Fall 2000 **CMSC 451: Homework 5** Clyde Kruskal

Due Thursday, December 14, 2000.

Problem 1. DIRECTED HAMILTONIAN PATH PROBLEM (DHP): given a directed simple graph, does it contain a path that starts at some vertex and goes to some other vertex, going through each remaining vertex exactly once.

DIRECTED HAMILTONIAN CYCLE PROBLEM (DHC): given a directed simple graph, does it contain a directed simple cycle that goes through each vertex exactly once.

Assume that the DIRECTED HAMILTONIAN PATH PROBLEM (DHP) is known to be NP-complete.

a. Show that the DIRECTED HAMILTONIAN CYCLE PROBLEM (DHC) is in NP.

b. Show that the DIRECTED HAMILTONIAN CYCLE PROBLEM (DHC) is complete for NP (that is, it is NP-hard).

Problem 2. Consider the optimization problem of finding the longest simple path in a directed graph (LSP), where the length of a path is the number of edges.

a. Define a decision version of the LSP problem.

b. Show that the decision version is in NP.

c. Show that the decision version is complete for NP (that is, it is NP-hard).

d. Show that if you could solve the optimization version in polynomial time that you could also solve the decision version in polynomial time.

e. Show that if you could solve the decision version in polynomial time that you could also solve the optimization version in polynomial time. HINT: First find the length of the longest simple path.

f. Show that you can solve the decision version for constant length paths in polynomial time.

Problem 3. Consider the gadget we used to prove that the hamiltonian cycle problem (HC) is NP-complete. Is there a "simpler" gadget that serves the same purpose? If so describe it, and justify. If not give a brief, informal explanation.

Problem 4. Do Excercise 37.2-2 on page 973 of CLR.

Problem 5. Do Excercise 37.2-3 on page 974 of CLR.