Exam #1 is Thursday 6-7:30pm Room CSIC 3117

Note Title 10/2/2007

You should review material going back to the first week.

For example:

- Rules for logs, calculus, sums, expected values, …
- Constructive induction, the Fibonacci examples and proofs…
- Algorithms that you've seen and now analyzed in more detail such as transitive closure, several sorts…
- Techniques to go from code or pseudocode to either summations or recurrences…
- The five asymptotic relationships, their quantified definitions, how to construct c and then n_0 for the ones that start with existential quantifiers, the limit-based definitions, how we can use these relationships to answer specific questions…
- Recurrence trees and how they can be used to get nice Big-O and Big-Omega bounds…

Recurrence Recap

Note Title 10/1/2007

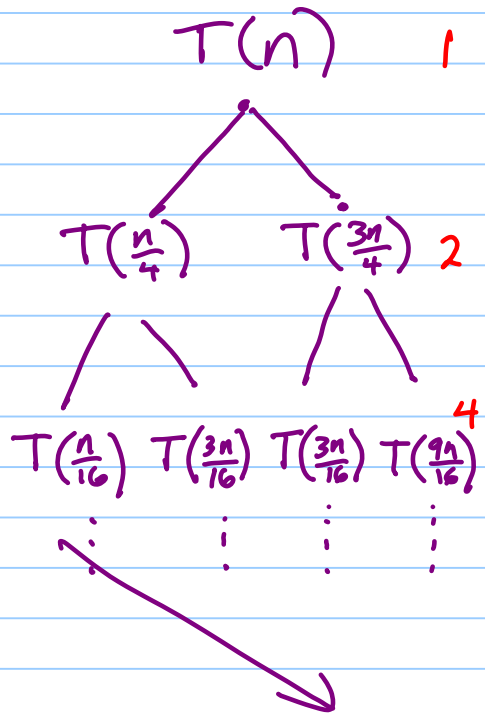
We are given some recurrence relation that doesn't fit into the Master Theorem well such as:

$T(1)=1$

$T(n)=T(n/4)+T(3n/4)+1$

Things we want to observe.

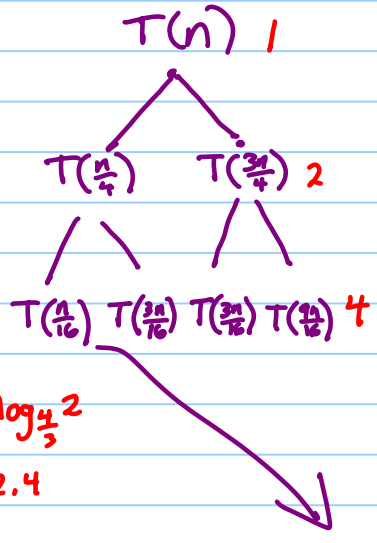
- Structure of the tree
  - Symmetrical?
  - Density?
  - Number of levels?
- Work done
  - At full interior levels?
  - At leaves?
- Asymptotic relationships
  - Big-Omega
  - Big-O



How many levels?

Work per interior level?

How many leaves?



Levels: $(\frac{3}{4})^{\text{levels}} n = 1 \Rightarrow \text{levels} = \log_{\frac{4}{3}} n$

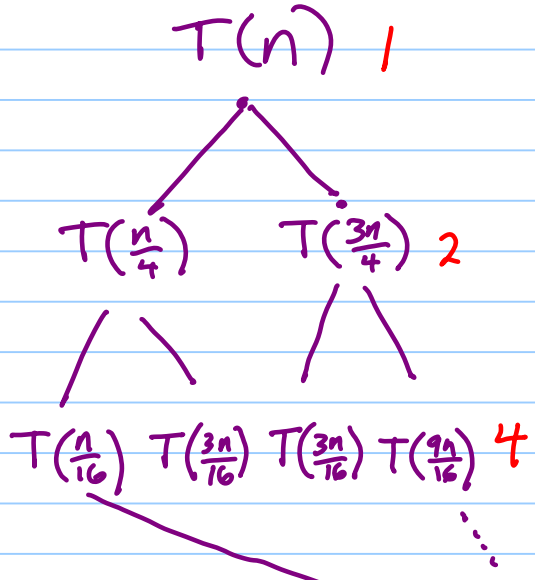
Work per interior level: 2^{level} (at most)

Number of leaves: $2^{\#\text{levels}} = 2^{\log_{\frac{4}{3}} n} = n^{\log_{\frac{4}{3}} 2} \approx n^{2.4}$ (at most)

Recall: $b^{\log a} = a^{\log b}$

Now Compute work for internal nodes (ignoring leaves) – Next page

What is the amount of work in the tree?



At most:

$$\leq \sum_{i=0}^{\log_{\frac{4}{3}} n - 1} 2^i =$$

$$= \frac{2^{\log_{\frac{4}{3}} n} - 1}{2 - 1}$$

$$= n^{\log_{\frac{4}{3}} 2} - 1$$

$$\simeq n^{2.4} - 1$$

At least:

$$\geq \sum_{i=0}^{\log_4 n - 1} 2^i =$$

$$= \frac{2^{\log_4 n} - 1}{2 - 1}$$

$$= n^{\log_4 2} - 1$$

$$= \sqrt{n} - 1$$

Recall:

$$\sum_{i=0}^{k} r^k$$

$$= \frac{r^{k+1} - 1}{r - 1}$$

What is the impact of the f(*n*)?

What if we make a change to only the f(n) in the previous recurrence:

T(1)=1

T(n)=T(n/4)+T(3n/4)+n