Some Helpful Math

Note Title 10/25/2007

$$\left(\frac{n}{2}\right)^n \in o(n^n)$$

Prove for ANY $c \geq 0$

$$\left(\frac{n}{2}\right)^n < cn^n$$

$$\frac{n^n}{2^n} < cn^n$$

$$n^n < cn^n 2^n$$

$$1 < c2^n$$

$$2^n > \frac{1}{c}$$

$$n > \log_2\left(\frac{1}{c}\right)$$

So.
For any $c > 0$
$n_0 = \log_2\left(\frac{1}{c}\right) + 1$
or 1,
whichever larger

Some facts about $n!$

① $n! = \sqrt{2\pi n}\left(\frac{n}{e}\right)^n e^{\alpha_n}$

where $\frac{1}{12n+1} < \alpha_n < \frac{1}{12n}$

② $n! \in o(n^n)$

③ $n! \in \omega(2^n)$