CMSC351 Final Exam Reference

Asymptotic Notations.

$\Theta(g(n)) = \{f(n)$: there exist positive constants c_1, c_2, and n_0 such that $0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n)$ for all $n \geq n_0\}$.

$O(g(n)) = \{f(n)$: there exist positive constants c and n_0 such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0\}$.

$\Omega(g(n)) = \{f(n)$: there exist positive constants c and n_0 such that $0 \leq cg(n) \leq f(n)$ for all $n \geq n_0\}$.

$f(n) = o(g(n))$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$.

$f(n) = \omega(g(n))$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} = \infty$.

$f(n) \sim g(n)$ if $f(n) = g(n) + o(g(n))$.

Logarithms.

$$\begin{aligned}
a &= b^{\log_b a} \\
\log_c(ab) &= \log_c a + \log_c b \\
\log_b a^n &= n \log_b a \\
\log_b a &= \frac{\log_c a}{\log_c b} \\
\log_b(1/a) &= -\log_b a \\
\log_b a &= \frac{1}{\log_a b} \\
a^{\log_b n} &= n^{\log_b a} \\
a^{f(n)} &= e^{f(n)\ln a} \\
(\log_a f(n))' &= \frac{f'(n)}{f(n)\ln a} \\
(a^{f(n)})' &= \ln a f'(n) a^{f(n)}
\end{aligned}$$

Quadratic Formula.

$$ax^2 + bx + c = 0 \quad \Rightarrow \quad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Summations.

Simple Arithmetic Series:

$$\sum_{k=1}^{n} k = 1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = 1 + 4 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = 1 + 8 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

General Arithmetic Series:

$$\sum_{k=m}^{n} k = 1 + 2 + \cdots + n = \frac{(n-m+1)(n+m)}{2}$$

Geometric series:

$$\sum_{k=0}^{n} x^k = 1 + x + +x^2 \cdots + x^n = \frac{x^{n+1}-1}{x-1} \qquad x \neq 1$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \qquad |x| < 1$$

Integration rules - for an increasing function $f(x)$:

$$\int_{a-1}^{b} f(s)ds \leq \sum_{i=a}^{b} f(i) \leq \int_{a}^{b+1} f(s)ds$$

for a decreasing function $g(x)$:

$$\int_{a}^{b+1} g(s)ds \leq \sum_{i=a}^{b} g(i) \leq \int_{a-1}^{b} g(s)ds$$

Recurrences.
"AMT: Ambitious Master Theorem":

$$T(n) = \begin{cases} aT(n/b) + cn^d & n > 1 \\ f & n = 1 \end{cases}$$

implies

$$T(n) = \begin{cases} \left(f + \frac{c}{ab^{-d}-1}\right) n^{\log_b a} - (\frac{cn^d}{ab^{-d}-1}) = \begin{cases} \Theta(n^{\log_b a}) & a > b^d \\ \Theta(n^d) & a < b^d \end{cases} \\ n^d(f + c\log_b n) = \Theta(n^d \log_b n) & a = b^d \end{cases}.$$

Miscelleneous.
Stirling's Approximation:

$$n! \geq \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$$