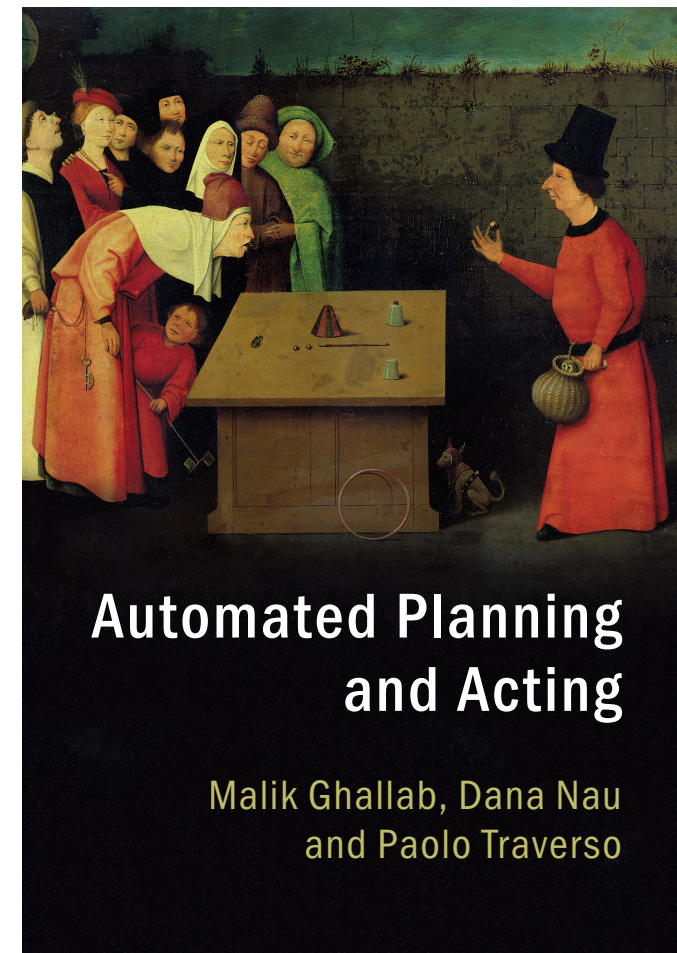


## Chapter 5

# Deliberation with Nondeterministic Domain Models

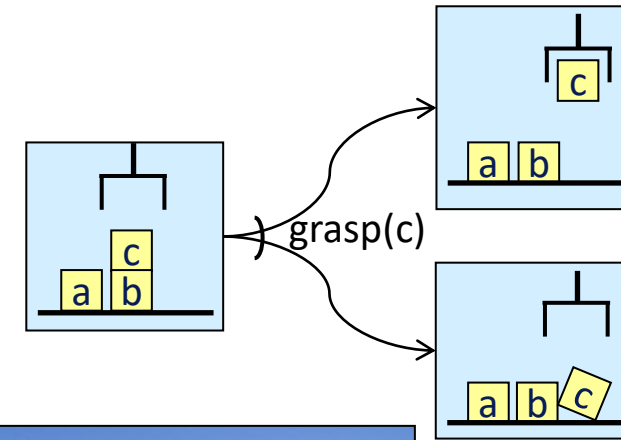
Dana S. Nau  
University of Maryland



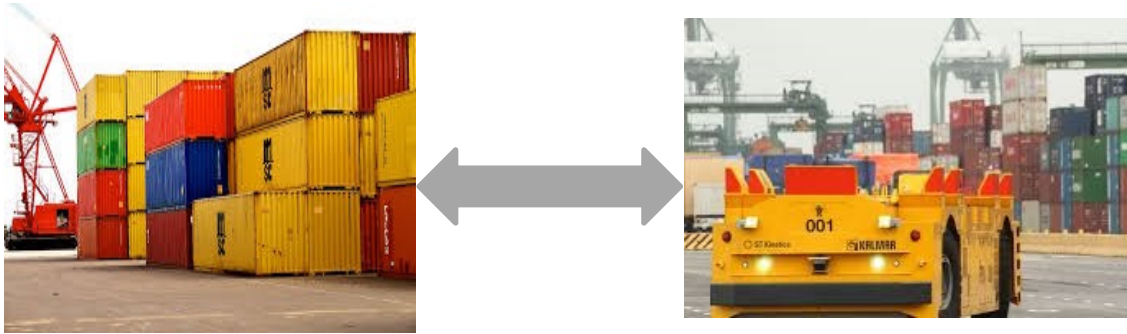
<http://www.laas.fr/planning>

# Motivation

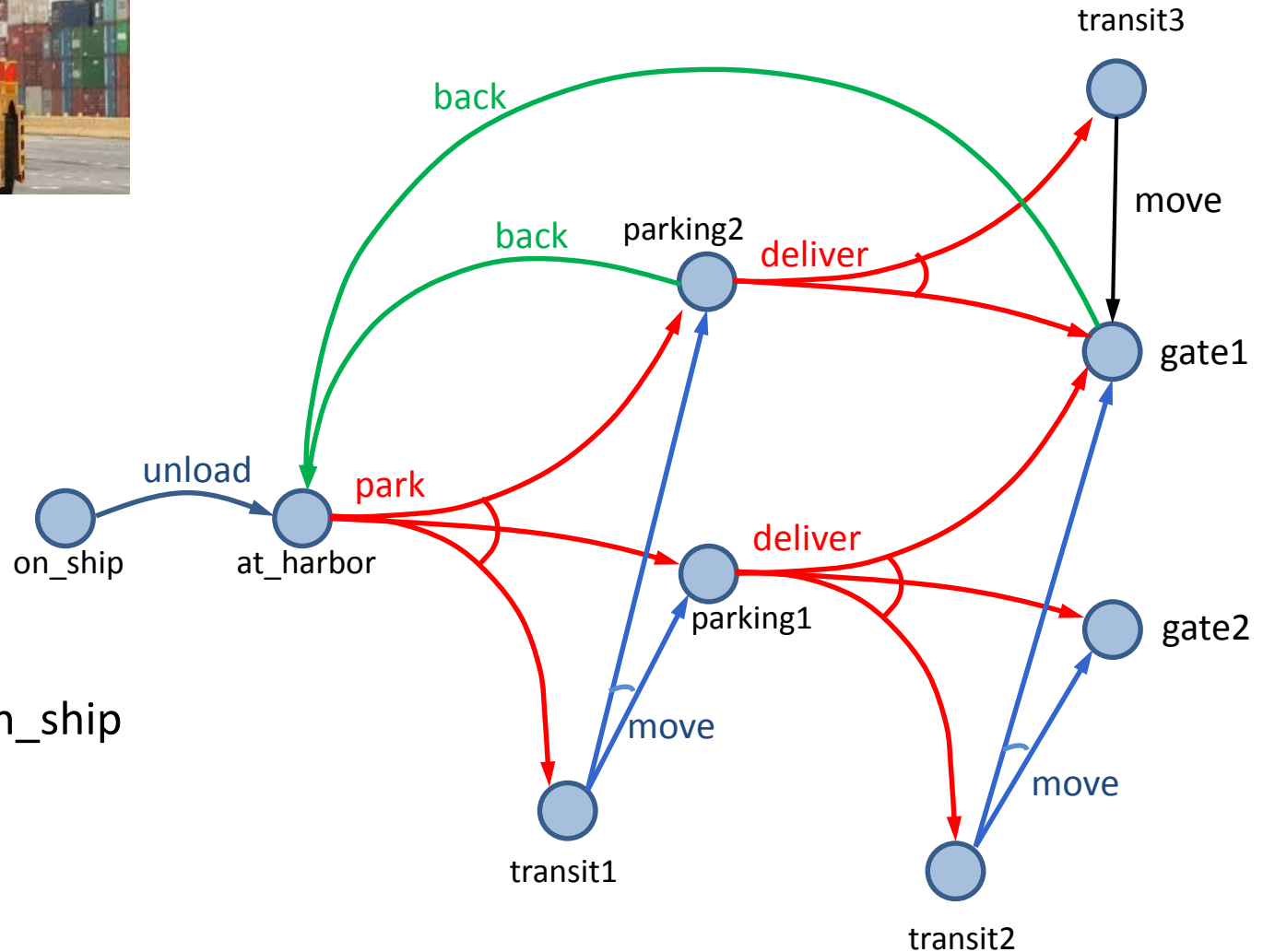
- We've assumed action  $a$  in state  $s$  has just one possible outcome
  - ▶  $\gamma(s, a)$
- Often more than one possible outcome
  - ▶ Unintended outcomes
  - ▶ Exogenous events
  - ▶ Inherent uncertainty



# Example

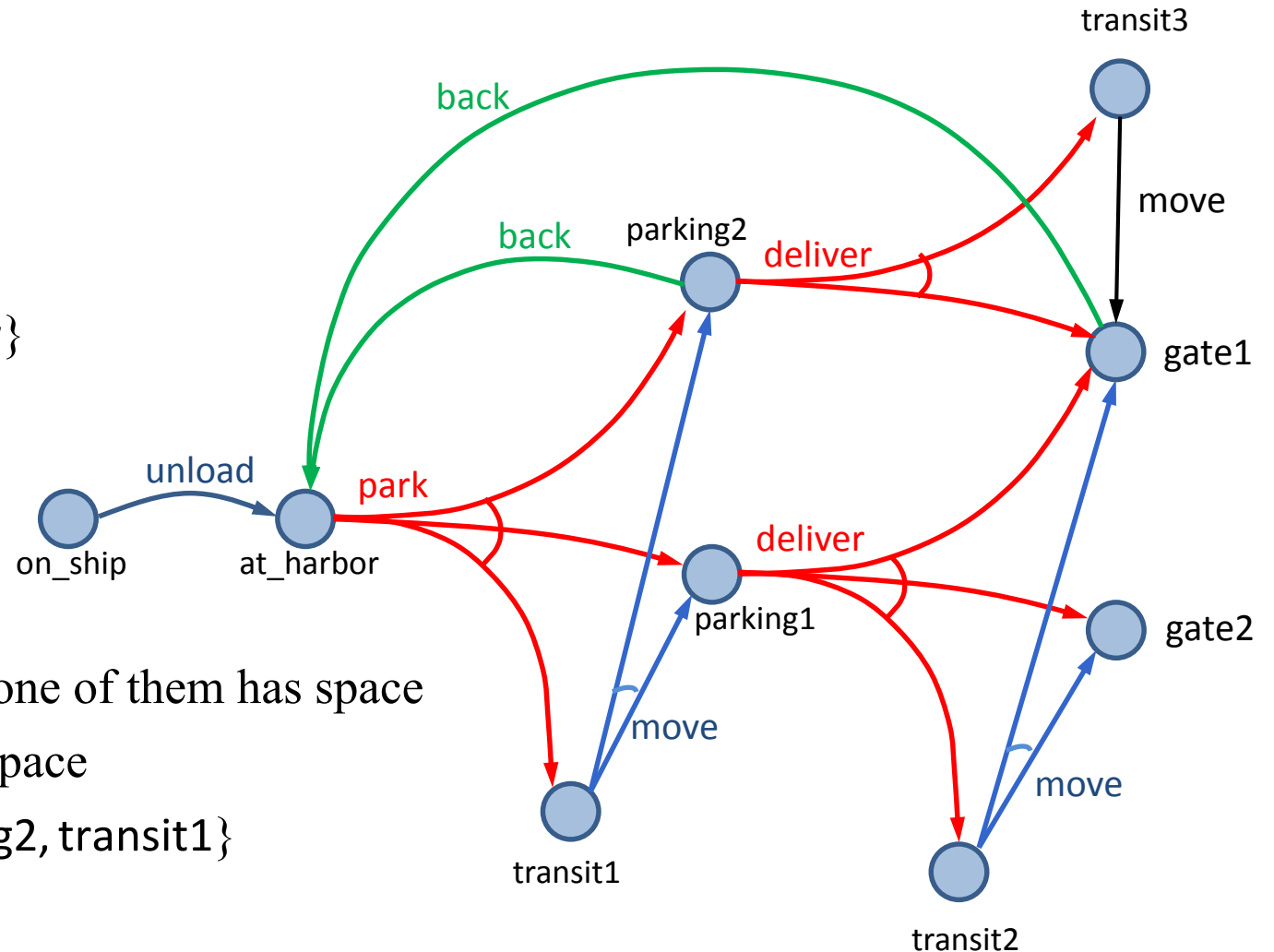


- Very simple harbor management domain
  - ▶ Unload a single item from a ship
  - ▶ Move it around a harbor
- One state variable:  $\text{pos}(\text{item})$ 
  - ▶ Simplified names for states
  - ▶ For  $\{\text{pos}(\text{item})=\text{on\_ship}\}$ , just write  $\text{on\_ship}$



# Nondeterministic Planning Domains

- 3-tuple  $(S, A, \gamma)$ 
  - ▶  $S$  and  $A$  – finite sets of states and actions
  - ▶  $\gamma: S \times A \rightarrow 2^S$
- $\gamma(s, a) = \{\text{all possible “next states” after applying action } a \text{ in state } s\}$ 
  - ▶  $a$  is applicable in state  $s$  iff  $\gamma(s, a) \neq \emptyset$
- $\text{Applicable}(s) = \{\text{all actions applicable in } s\}$   
 $= \{a \in A \mid \gamma(s, a) \neq \emptyset\}$
- Example:
  - ▶  $\text{Applicable}(\text{at\_harbor}) = \{\text{park}\}$
  - ▶ park has three possible outcomes
    - put item in parking1 or parking2 if one of them has space
    - or in transit1 if there’s no parking space
  - ▶  $\gamma(\text{at\_harbor}, \text{park}) = \{\text{parking1}, \text{parking2}, \text{transit1}\}$



# Nondeterministic Planning Domains

- One possible action representation:
  - ▶ like classical, but with  $n$  mutually exclusive “effects” lists

- e.g., park:

pre:  $\text{pos}(\text{item}) = \text{at\_harbor}$

eff<sub>1</sub>:  $\text{pos}(\text{item}) \leftarrow \text{parking1}$

eff<sub>2</sub>:  $\text{pos}(\text{item}) \leftarrow \text{parking2}$

eff<sub>3</sub>:  $\text{pos}(\text{item}) \leftarrow \text{transit1}$

- Problem:

- ▶ number of effects lists may be combinatorially large

- ▶ Suppose  $a$  can cause any possible combination of effects  $e_1, e_2, \dots, e_k$

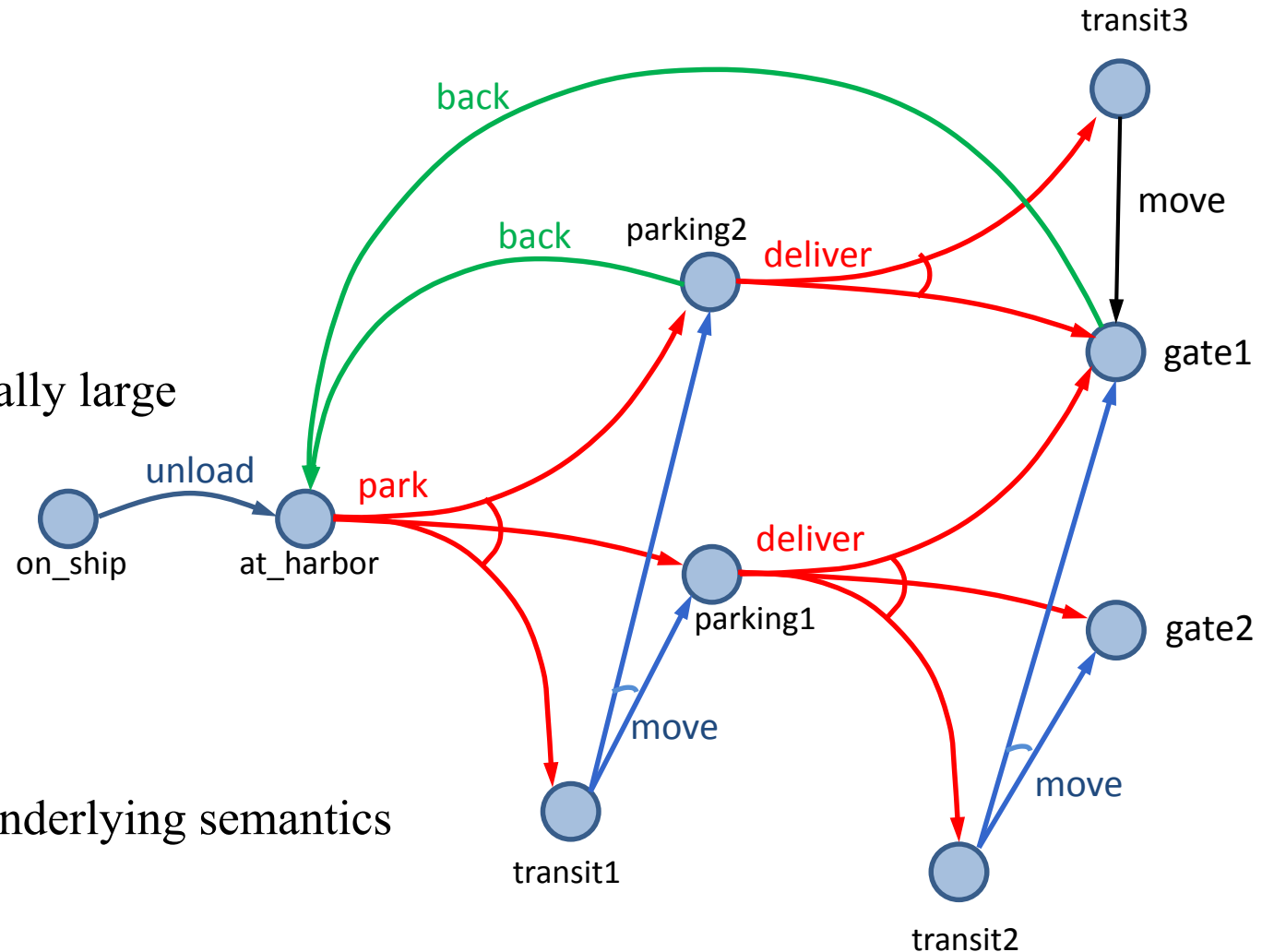
- ▶ Need  $\text{eff}_1, \text{eff}_2, \dots, \text{eff}_{2^k}$

- One for each combination

- ▶ Section 5.4: a way to alleviate this

- For now, ignore most of that, just look at the underlying semantics

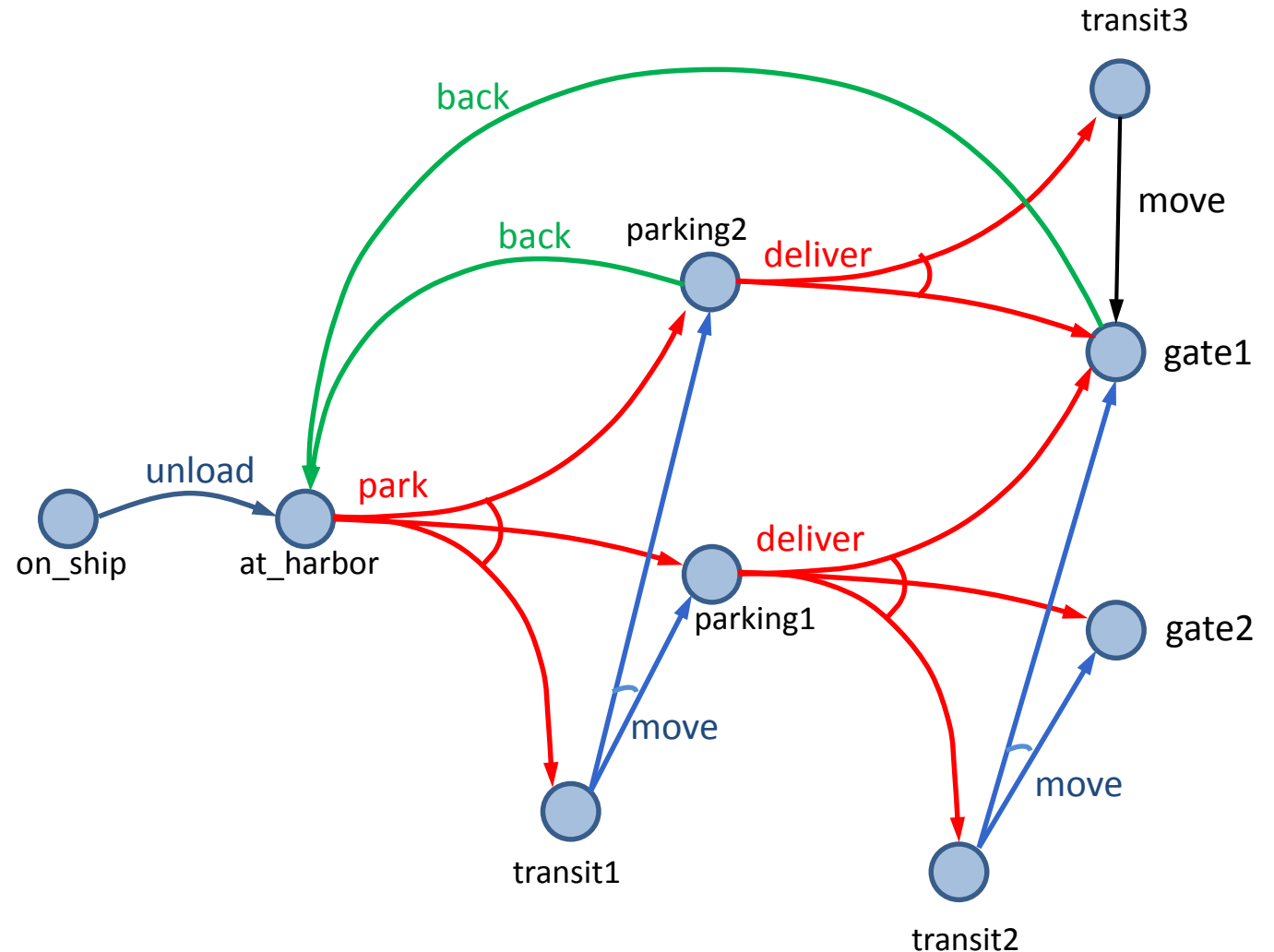
- ▶ states, actions  $\Leftrightarrow$  nodes, edges in a graph





# Nondeterministic Planning Domains

- For deterministic planning problems, search space was a graph
- Now it's an AND/OR graph
  - ▶ *OR branch*:
    - several applicable actions, which one to choose?
  - ▶ *AND branch*:
    - multiple possible outcomes, must handle all of them
- Analogy to PSP in Chapter 2
  - ▶ *OR branch*  $\Leftrightarrow$  action selection
  - ▶ *AND branch*  $\Leftrightarrow$  flaw selection



# Plans Policies

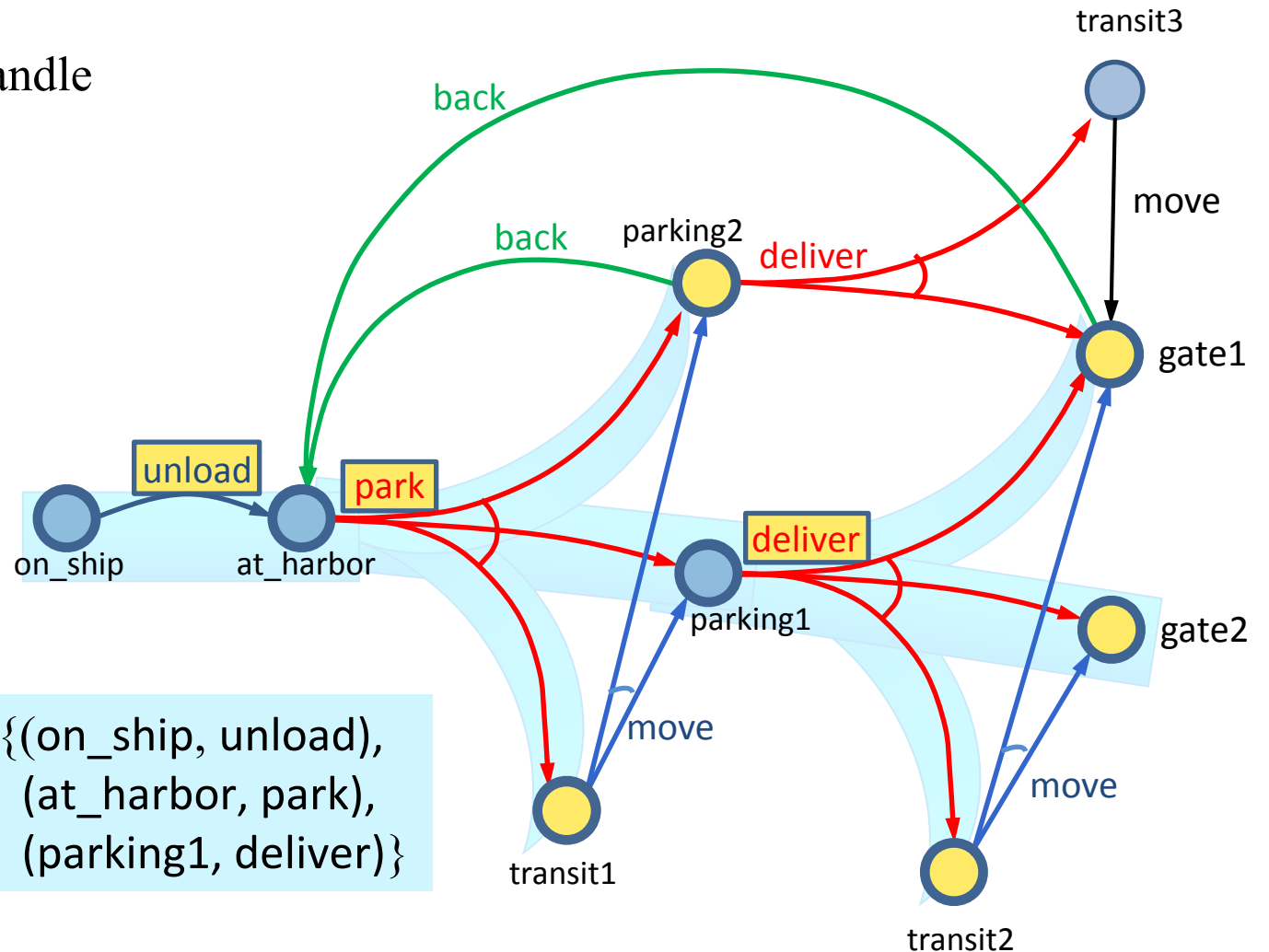
- Need something more general than a sequence of actions (unload, park, deliver)
  - ▶ After park, what do we do next?
    - multiple possible outcomes, must handle all of them
- *Policy*: a partial function  $\pi : S \rightarrow A$ 
  - ▶ i.e.,  $\text{Dom}(\pi) \subseteq S$
  - ▶ For every  $s \in \text{Dom}(\pi)$ , require  $\pi(s) \in \text{Applicable}(s)$

- Meaning: perform  $\pi(s)$  whenever we're in state  $s$
- Two equivalent notations:

$\pi_1(\text{on\_ship}) = \text{unload}$ ,  
 $\pi_1(\text{at\_harbor}) = \text{park}$ ,  
 $\pi_1(\text{parking1}) = \text{deliver}$

$\pi_1 = \{(\text{on\_ship}, \text{unload}),$   
 $(\text{at\_harbor}, \text{park}),$   
 $(\text{parking1}, \text{deliver})\}$

- That's just the notation
  - ▶ implementation could be quite different



# Definitions Over Policies

- *Transitive closure*:
  - ▶  $\hat{\gamma}(s, \pi) = \{\text{all states reachable from } s \text{ using } \pi\}$

▶  $\hat{\gamma}(s, \pi) = S_0 \cup S_1 \cup S_2 \cup \dots$

$S_0 = \{s\}$

$S_1 = S_0 \cup \{\gamma(s_0, \pi(s_0)) \mid s_0 \in S_0\}$

$S_2 = S_1 \cup \{\gamma(s_1, \pi(s_1)) \mid s_1 \in S_1\}$

...

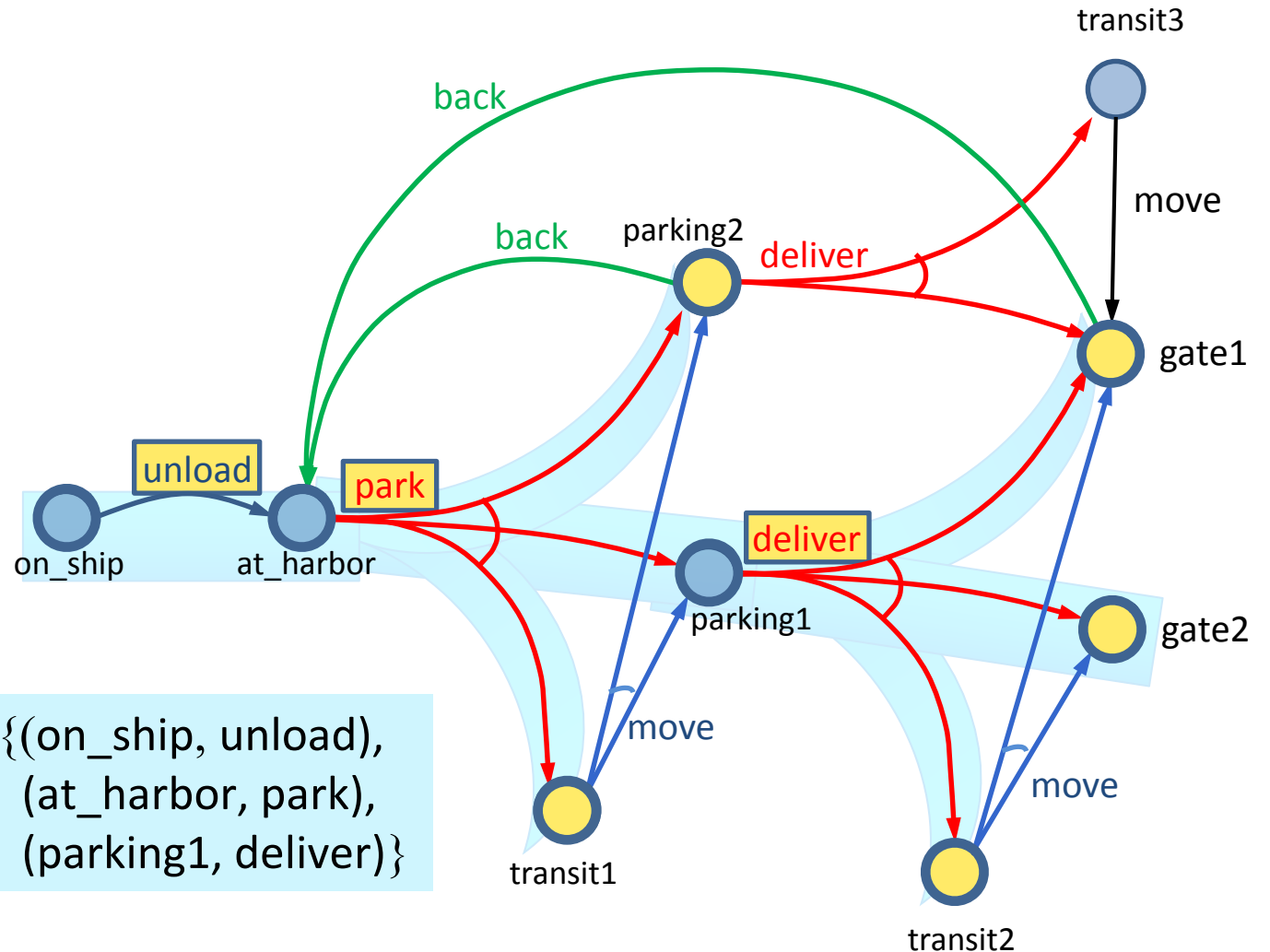
- *Reachability graph*:  $\text{Graph}(s, \pi) = (V, E)$

▶  $V = \hat{\gamma}(s, \pi)$

▶  $E = \{(s_1, s_2) \mid s_1 \in V, s_2 \in \gamma(s_1, \pi(s_1))\}$

- $\text{leaves}(s, \pi) = \hat{\gamma}(s, \pi) \setminus \text{Dom}(\pi)$

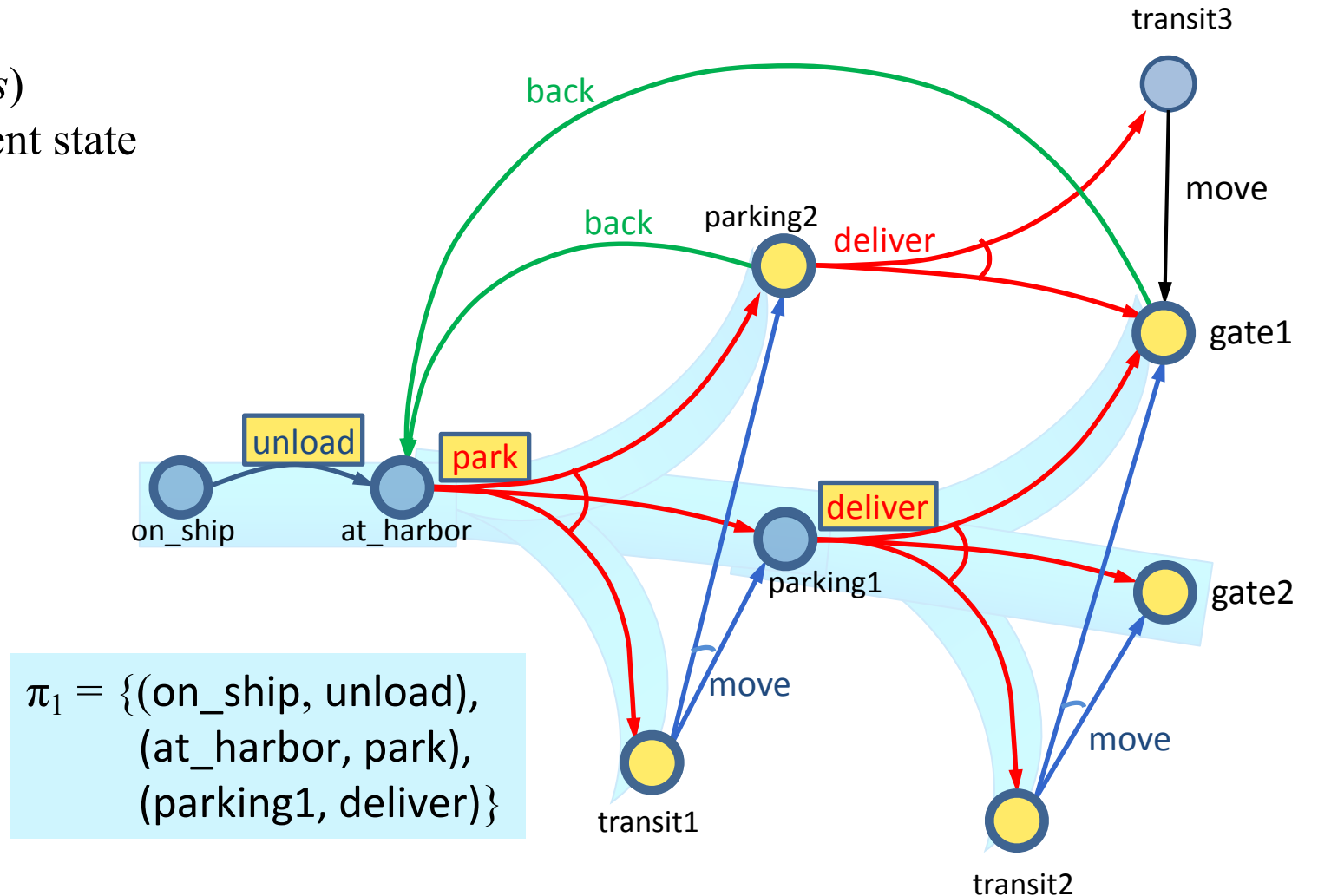
- ▶ may be empty





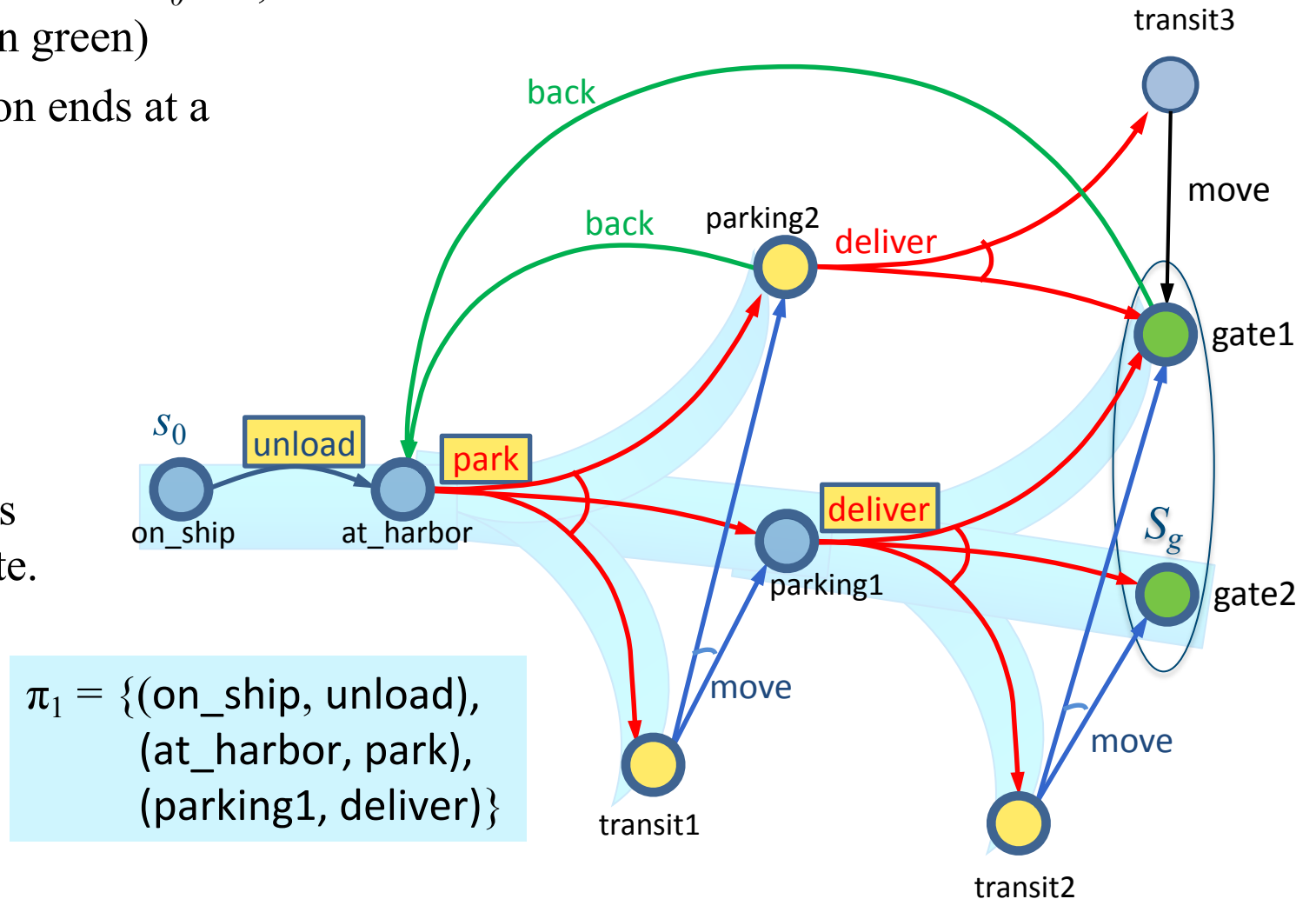
# Performing a Policy

- PerformPolicy( $\pi$ )
  - $s \leftarrow$  observe current state
  - while  $s \in \text{Dom}(\pi)$  do
  - perform action  $\pi(s)$
  - $s \leftarrow$  observe current state



# Planning Problems and Solutions

- Planning problem  $P = (\Sigma, s_0, S_g)$ 
  - ▶ planning domain  $\Sigma = (S, A, \gamma)$ , initial state  $s_0 \in S$ , set of goal states  $S_g \subseteq S$  (shown in green)
- $\pi$  is a *solution* if at least one execution ends at a goal
  - ▶  $leaves(s, \pi) \cap S_g \neq \emptyset$
- A solution  $\pi$  is *safe* if
  - $\forall s \in \hat{\gamma}(s_0, \pi), leaves(s, \pi) \cap S_g \neq \emptyset$
  - ▶ at every state in  $\hat{\gamma}(s_0, \pi)$ , at least one of the execution paths from  $s$  using  $\pi$  stops at a goal state.
- Otherwise, *unsafe* solution
  - ▶ Is  $\pi_1$  safe or unsafe?

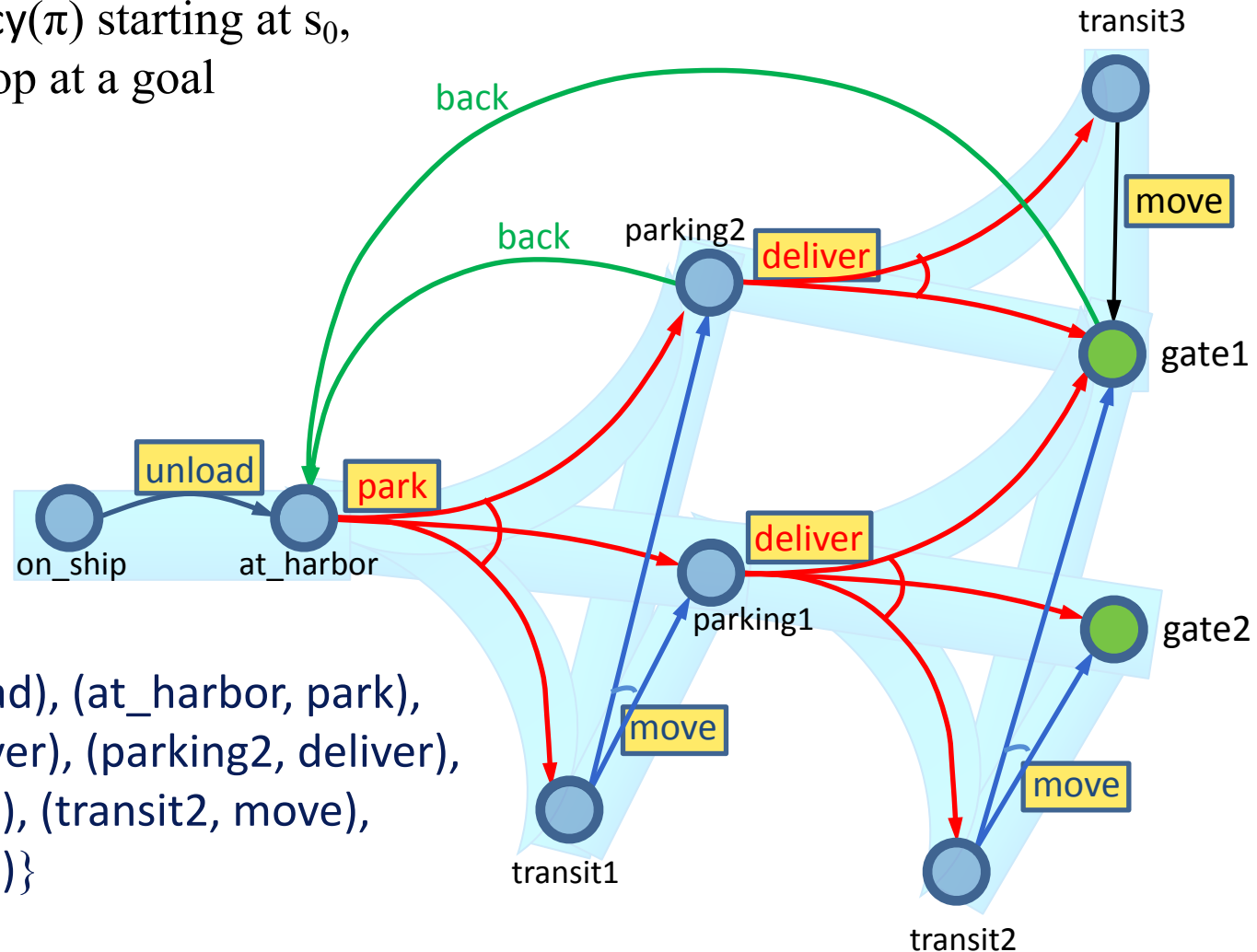


# Safe Solutions

- *Acyclic* safe solution
  - $\text{Graph}(s_0, \pi)$  is acyclic, and  $\text{leaves}(s, \pi) \subseteq S_g$
- If we run  $\text{PerformPolicy}(\pi)$  starting at  $s_0$ , we're guaranteed to stop at a goal

```

    • PerformPolicy( $\pi$ )
       $s \leftarrow$  observe current state
      while  $s \in \text{Dom}(\pi)$  do
        perform action  $\pi(s)$ 
         $s \leftarrow$  observe current state
    
```



$\pi_2 = \{(\text{on\_ship}, \text{unload}), (\text{at\_harbor}, \text{park}), (\text{parking1}, \text{deliver}), (\text{parking2}, \text{deliver}), (\text{transit1}, \text{move}), (\text{transit2}, \text{move}), (\text{transit3}, \text{move})\}$

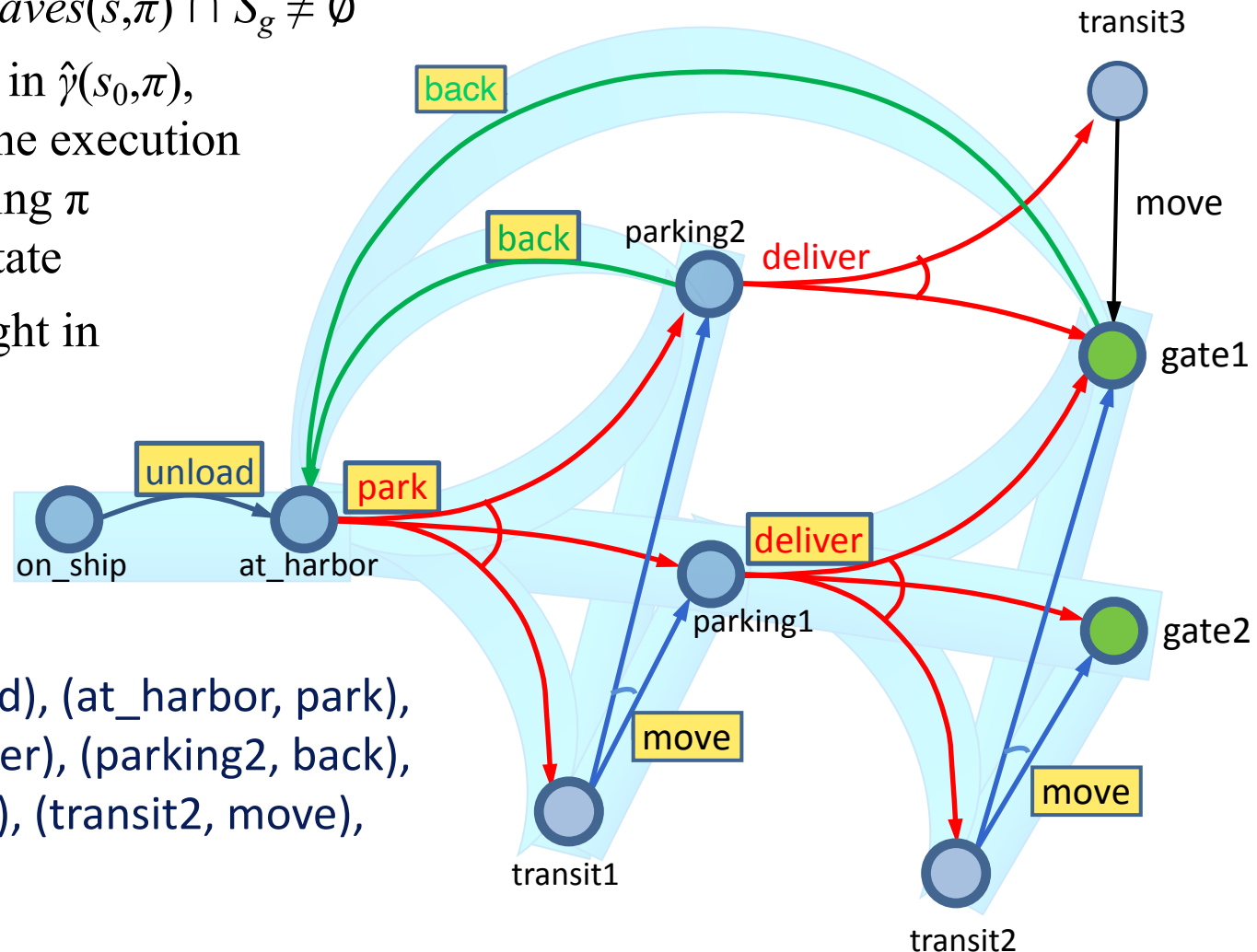
# Safe Solutions

- *Cyclic* safe solution

- ▶  $\text{Graph}(s_0, \pi)$  is cyclic, and  $\text{leaves}(s, \pi) \subseteq S_g$ ,  
and  $\forall s \in \hat{\gamma}(s_0, \pi), \text{leaves}(s, \pi) \cap S_g \neq \emptyset$

- At every state  $s$  in  $\hat{\gamma}(s_0, \pi)$ ,  
at least one of the execution  
paths from  $s$  using  $\pi$   
ends at a goal state
- ▶ Will never get caught in  
a dead end

$\pi_3 = \{(\text{on\_ship}, \text{unload}), (\text{at\_harbor}, \text{park}),$   
 $(\text{parking1}, \text{deliver}), (\text{parking2}, \text{back}),$   
 $(\text{transit1}, \text{move}), (\text{transit2}, \text{move}),$   
 $(\text{gate1}, \text{back})\}$



- $\text{PerformPolicy}(\pi)$ 
  - $s \leftarrow$  observe current state
  - while  $s \in \text{Dom}(\pi)$  do
  - perform action  $\pi(s)$
  - $s \leftarrow$  observe current state

**Poll:** Let  $\pi$  be a cyclic safe solution. Suppose we run  $\text{PerformPolicy}(\pi)$  starting at  $s_0$ .

1. Are there situations where we can be sure  $\pi$  will reach a goal?
2. Are there situations where we can't be sure  $\pi$  will reach a goal?

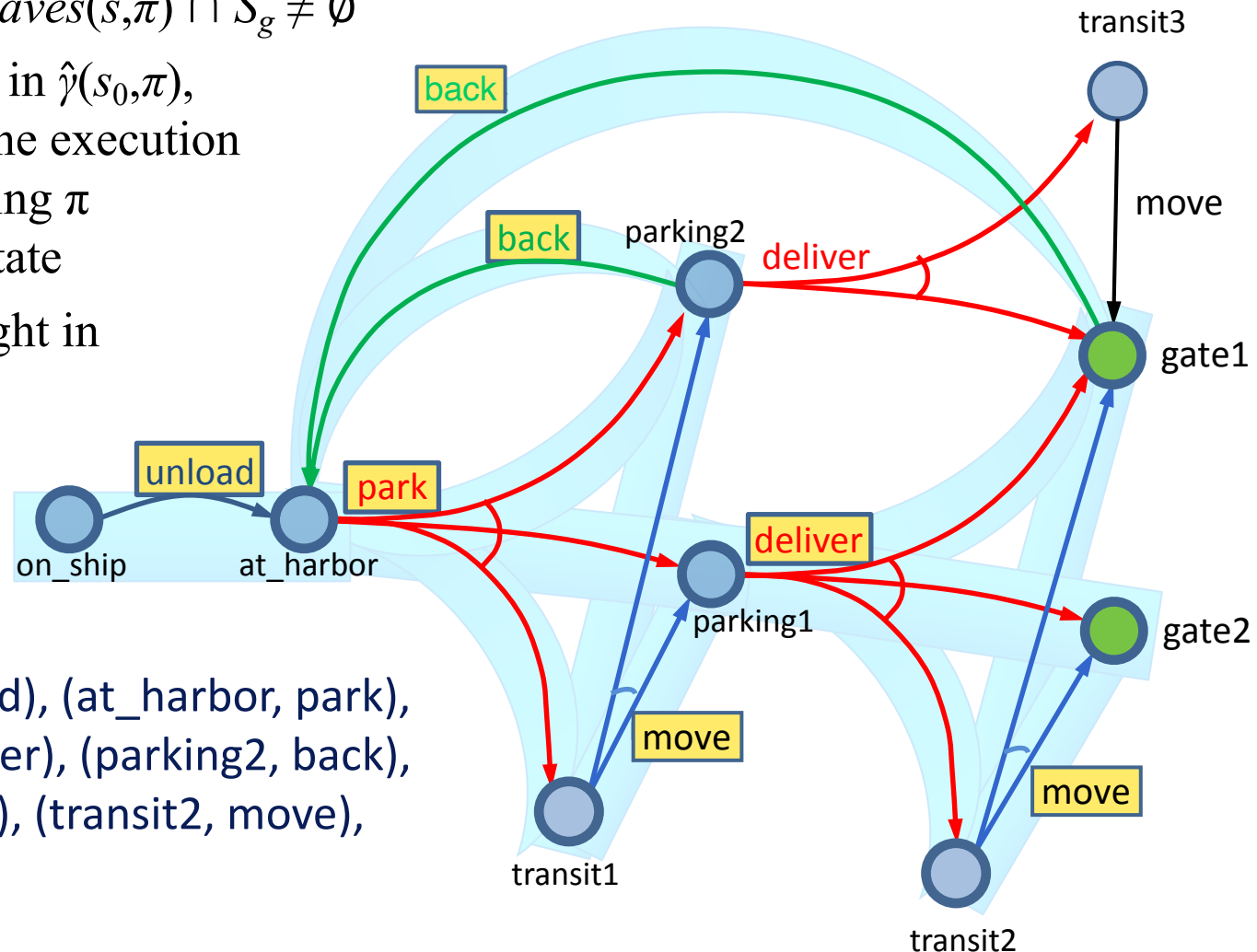
# Safe Solutions

- *Cyclic* safe solution

- ▶  $\text{Graph}(s_0, \pi)$  is cyclic, and  $\text{leaves}(s, \pi) \subseteq S_g$ , and  $\forall s \in \hat{\gamma}(s_0, \pi), \text{leaves}(s, \pi) \cap S_g \neq \emptyset$

- At every state  $s$  in  $\hat{\gamma}(s_0, \pi)$ , at least one of the execution paths from  $s$  using  $\pi$  ends at a goal state
- ▶ Will never get caught in a dead end
- ▶ Every “fair” execution will reach a goal

$\pi_3 = \{(\text{on\_ship}, \text{unload}), (\text{at\_harbor}, \text{park}), (\text{parking1}, \text{deliver}), (\text{parking2}, \text{back}), (\text{transit1}, \text{move}), (\text{transit2}, \text{move}), (\text{gate1}, \text{back})\}$

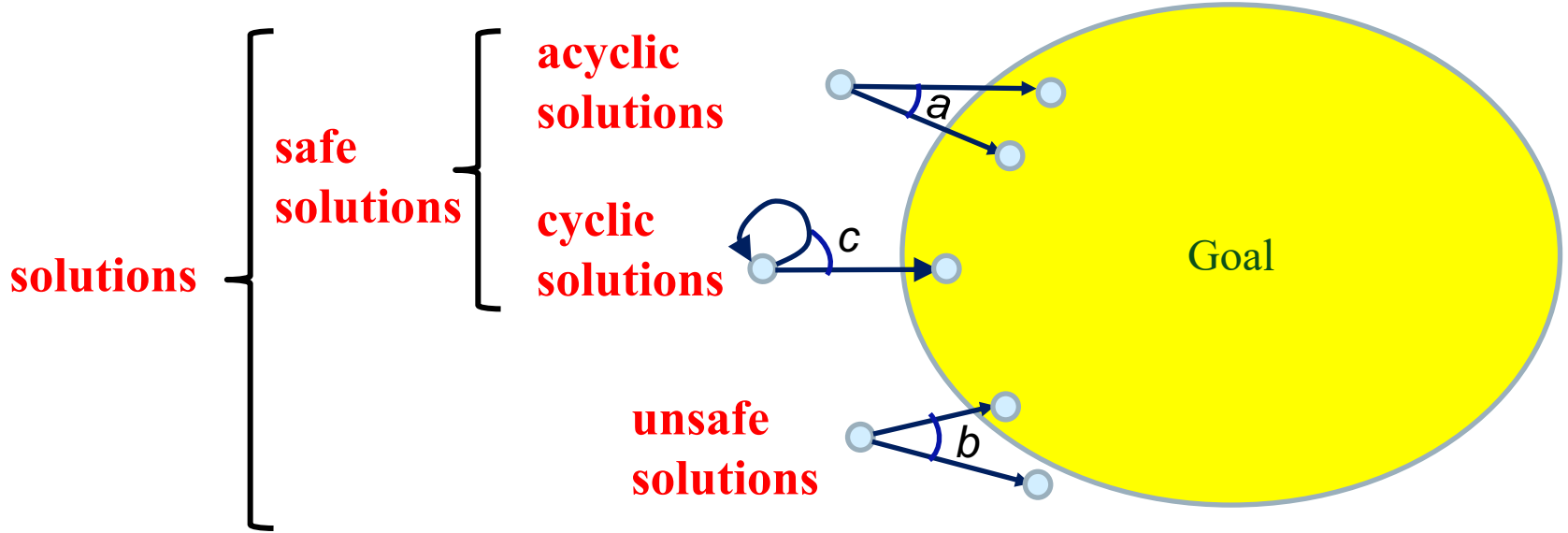


- $\text{PerformPolicy}(\pi)$ 
  - $s \leftarrow \text{observe current state}$
  - while  $s \in \text{Dom}(\pi)$  do
  - perform action  $\pi(s)$
  - $s \leftarrow \text{observe current state}$

## Poll:

1. Can you think of a real-world situation in which all executions are “fair”?
2. Can you think of a real-world situation in which there are “unfair” executions?

# Kinds of Solutions





# Finding (Unsafe) Solutions

For comparison:

Forward-search  $(\Sigma, s_0, g)$

$s \leftarrow s_0; \pi \leftarrow \langle \rangle$

loop

if  $s$  satisfies  $g$  then return  $\pi$

$A' \leftarrow \{a \in A \mid a \text{ is applicable in } s\}$

if  $A' = \emptyset$  then return failure

nondeterministically choose  $a \in A'$

$s \leftarrow \gamma(s, a); \pi \leftarrow \pi.a$

Find-Solution  $(\Sigma, s_0, S_g)$

$\pi \leftarrow \emptyset; s \leftarrow s_0; Visited \leftarrow \{s_0\}$

loop

if  $s \in S_g$  then return  $\pi$

$A' \leftarrow \text{Applicable}(s)$

if  $A' = \emptyset$  then return failure

nondeterministically choose  $a \in A'$

(\*) nondeterministically choose  $s' \in \gamma(s, a)$

if  $s' \in Visited$  then return failure

$\pi(s) \leftarrow a; Visited \leftarrow Visited \cup \{s'\}; s \leftarrow s'$

Decide which state to plan for

Cycle-checking

**Poll:** which should (\*) be?

A. nondeterministically choose

B. arbitrarily choose

# Example

Find-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$ ;  $s \leftarrow s_0$ ;  $Visited \leftarrow \{s_0\}$

loop

if  $s \in S_g$  then return  $\pi$

$A' \leftarrow \text{Applicable}(s)$

if  $A' = \emptyset$  then return failure

nondeterministically choose  $a \in A'$

nondeterministically choose  $s' \in \gamma(s, a)$

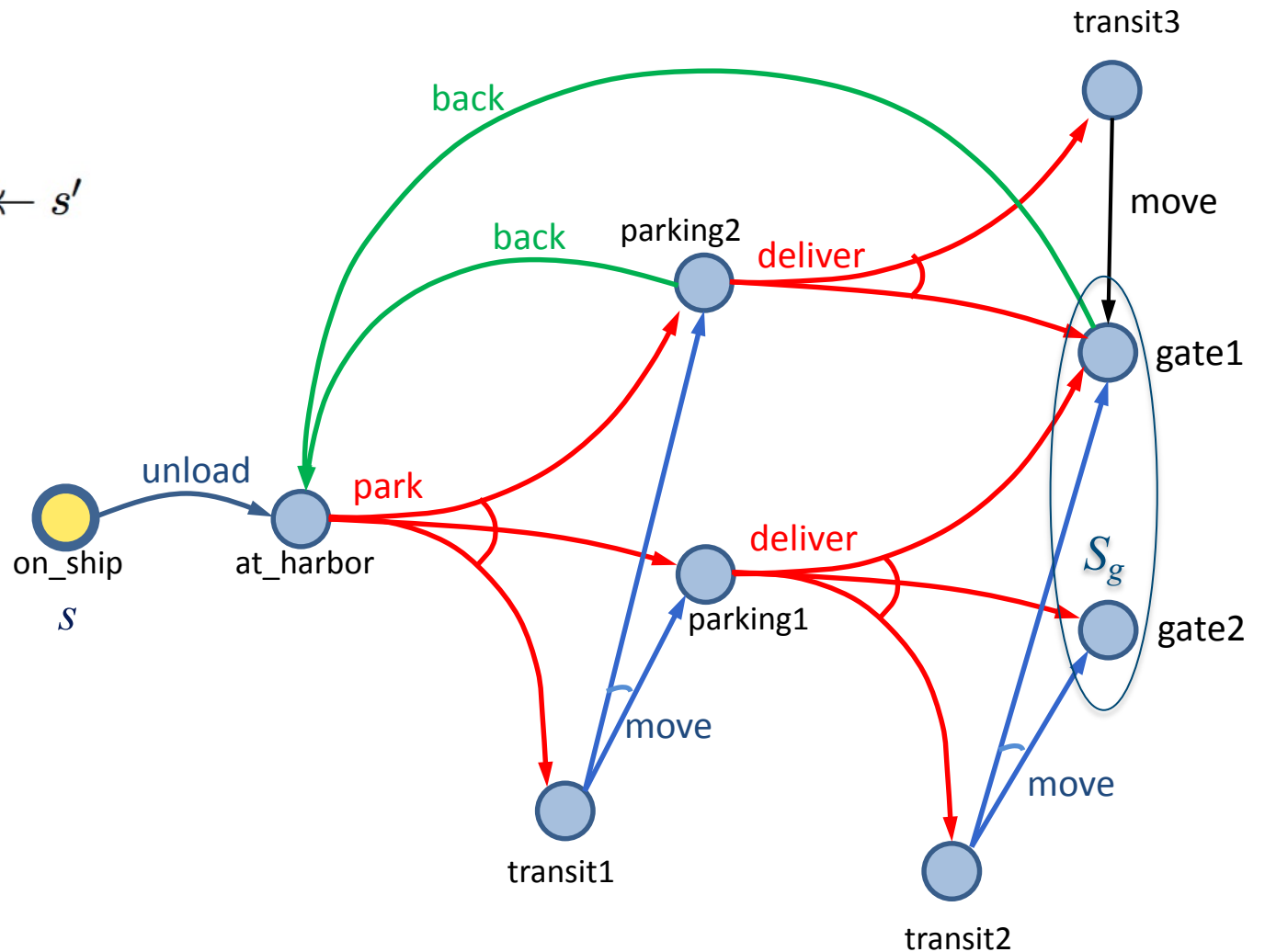
if  $s' \in Visited$  then return failure

$\pi(s) \leftarrow a$ ;  $Visited \leftarrow Visited \cup \{s'\}$ ;  $s \leftarrow s'$

$s = \text{on\_ship}$

$\pi = \{\}$

$Visited = \{\text{on\_ship}\}$



# Example

Find-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$ ;  $s \leftarrow s_0$ ;  $Visited \leftarrow \{s_0\}$

loop

if  $s \in S_g$  then return  $\pi$

$A' \leftarrow \text{Applicable}(s)$

if  $A' = \emptyset$  then return failure

nondeterministically choose  $a \in A'$

nondeterministically choose  $s' \in \gamma(s, a)$

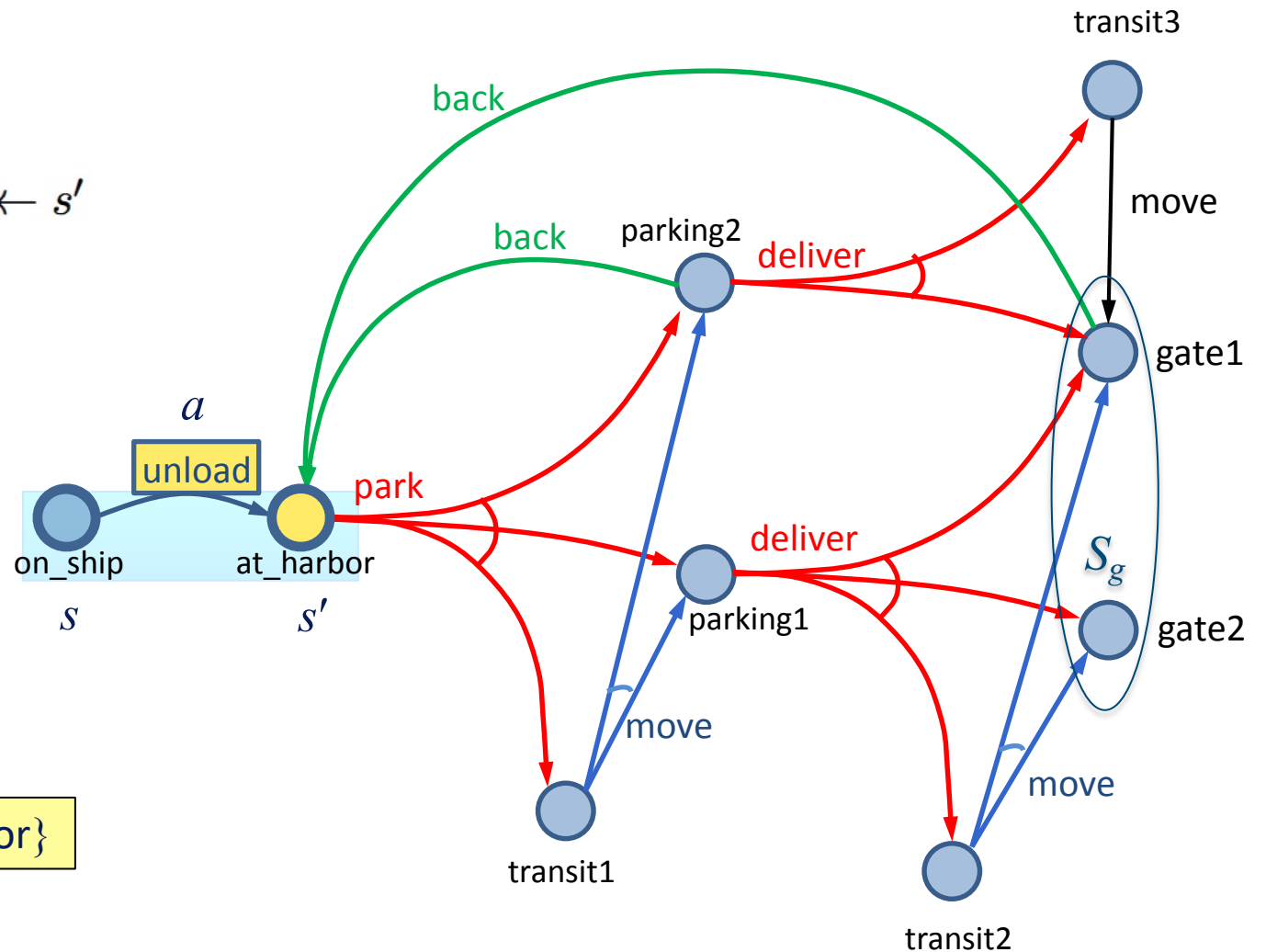
if  $s' \in Visited$  then return failure

$\pi(s) \leftarrow a$ ;  $Visited \leftarrow Visited \cup \{s'\}$ ;  $s \leftarrow s'$

$s = \text{on\_ship}$   
 $a = \text{unload}$   
 $\gamma(s, a) = \{\text{at\_harbor}\}$   
 $s \leftarrow s' = \text{at\_harbor}$

$\pi = \{(\text{on\_ship}, \text{unload})\}$

$Visited = \{\text{on\_ship}, \text{at\_harbor}\}$



# Example

Find-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$ ;  $s \leftarrow s_0$ ;  $Visited \leftarrow \{s_0\}$

loop

if  $s \in S_g$  then return  $\pi$

$A' \leftarrow \text{Applicable}(s)$

if  $A' = \emptyset$  then return failure

nondeterministically choose  $a \in A'$

nondeterministically choose  $s' \in \gamma(s, a)$

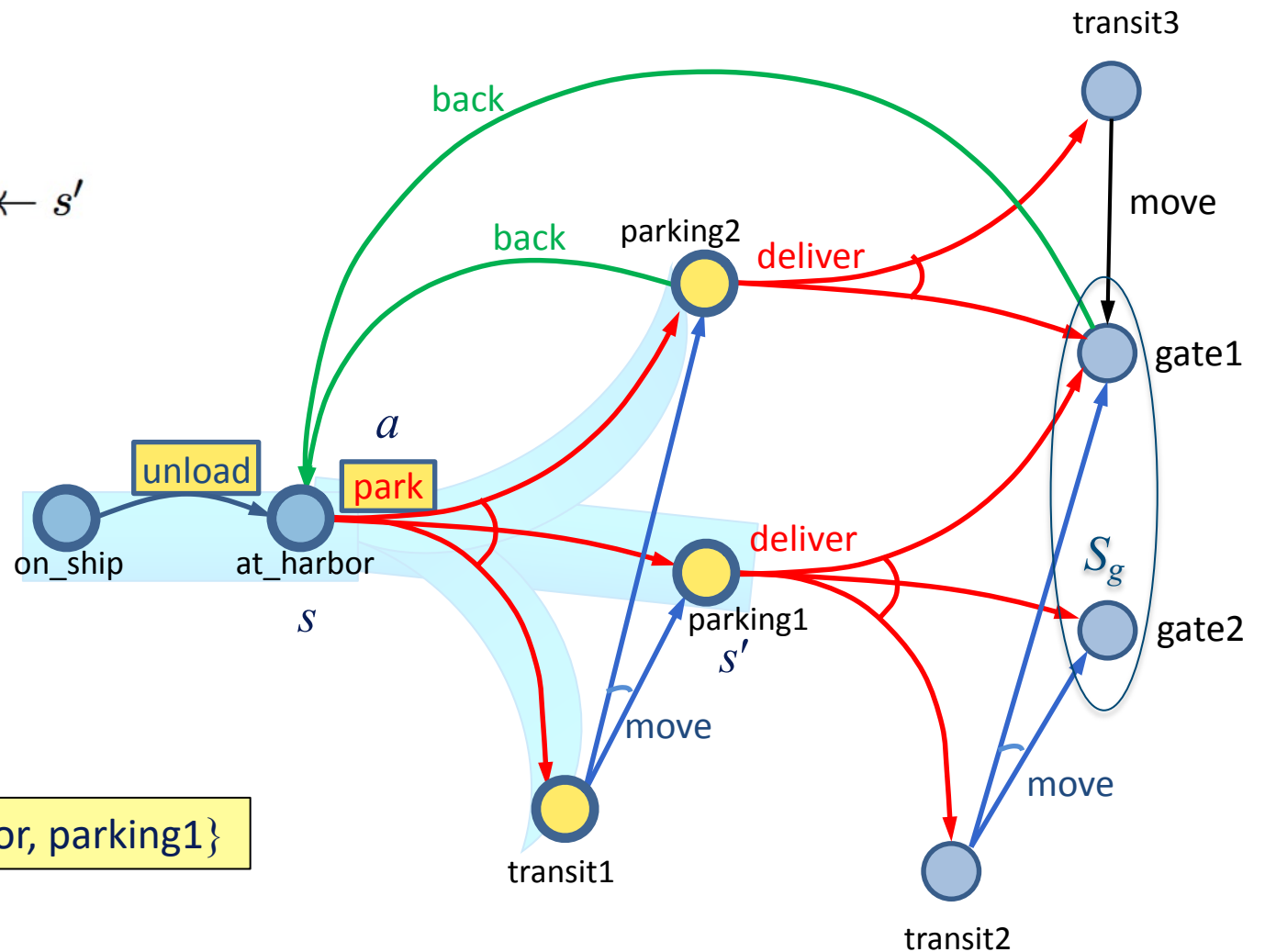
if  $s' \in Visited$  then return failure

$\pi(s) \leftarrow a$ ;  $Visited \leftarrow Visited \cup \{s'\}$ ;  $s \leftarrow s'$

$s = \text{at\_harbor}$   
 $a = \text{park}$   
 $\gamma(s, a) = \{\text{parking1},$   
 $\text{parking2},$   
 $\text{transit1}\}$   
 $s \leftarrow s' = \text{parking1}$

$\pi = \{(\text{on\_ship}, \text{unload}),$   
 $(\text{at\_harbor}, \text{park})\}$

$Visited = \{\text{on\_ship}, \text{at\_harbor}, \text{parking1}\}$



# Example

Find-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$ ;  $s \leftarrow s_0$ ;  $Visited \leftarrow \{s_0\}$

loop

if  $s \in S_g$  then return  $\pi$

$A' \leftarrow \text{Applicable}(s)$

if  $A' = \emptyset$  then return failure

nondeterministically choose  $a \in A'$

nondeterministically choose  $s' \in \gamma(s, a)$

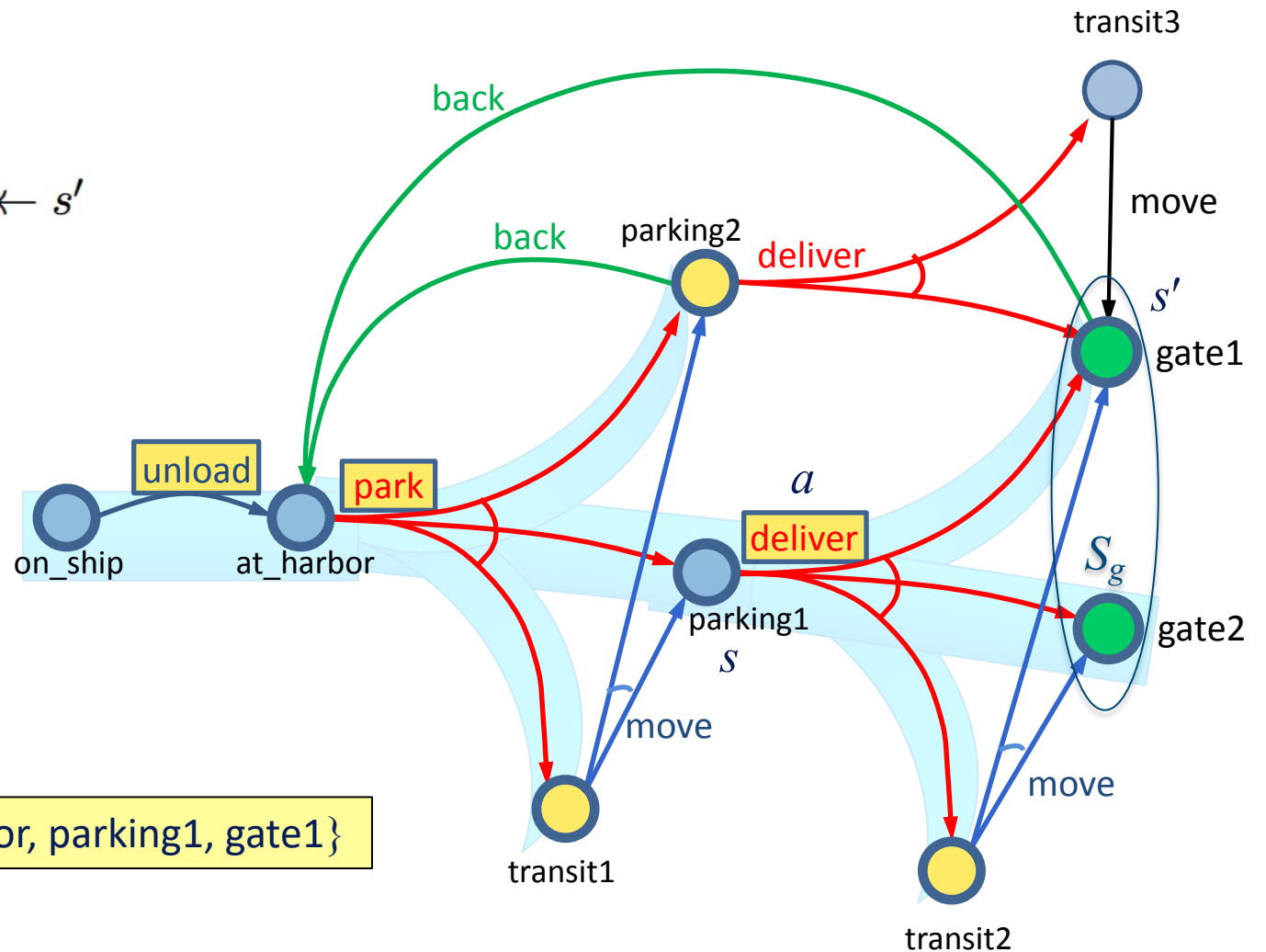
if  $s' \in Visited$  then return failure

$\pi(s) \leftarrow a$ ;  $Visited \leftarrow Visited \cup \{s'\}$ ;  $s \leftarrow s'$

$s = \text{parking1}$   
 $a = \text{deliver}$   
 $\gamma(s, a) = \{\text{gate1}, \text{gate2}, \text{transit2}\}$   
 $s \leftarrow s' = \text{gate1}$

$\pi = \{(\text{on\_ship}, \text{unload}), (\text{at\_harbor}, \text{park}), (\text{parking1}, \text{deliver})\}$

$Visited = \{\text{on\_ship}, \text{at\_harbor}, \text{parking1}, \text{gate1}\}$





# Example

Find-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$ ;  $s \leftarrow s_0$ ;  $Visited \leftarrow \{s_0\}$

loop

if  $s \in S_g$  then return  $\pi$

$A' \leftarrow \text{Applicable}(s)$

if  $A' = \emptyset$  then return failure

nondeterministically choose  $a \in A'$

nondeterministically choose  $s' \in \gamma(s, a)$

if  $s' \in Visited$  then return failure

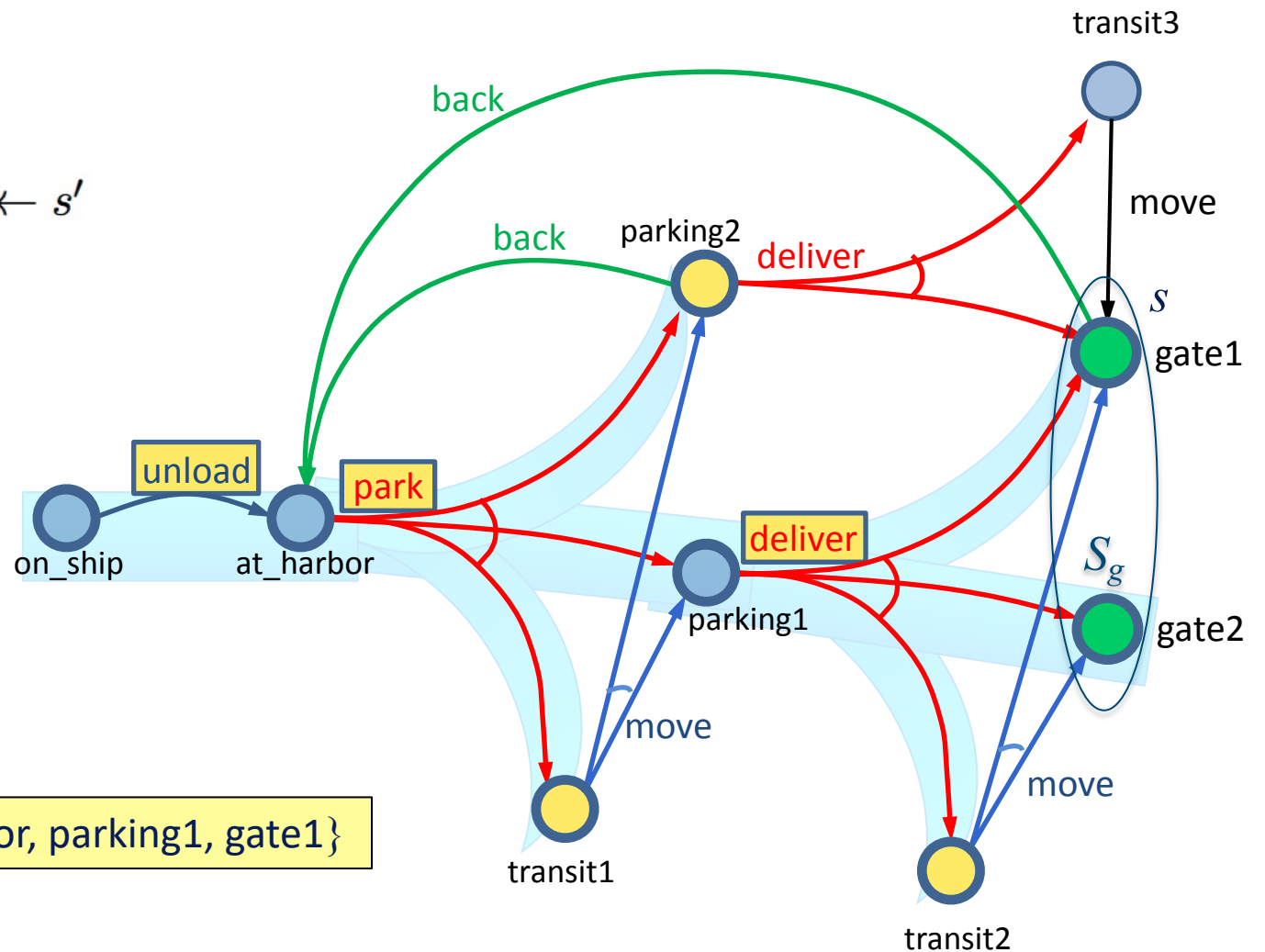
$\pi(s) \leftarrow a$ ;  $Visited \leftarrow Visited \cup \{s'\}$ ;  $s \leftarrow s'$

$s = \text{gate1}$

gate1 is a goal,  
so return  $\pi$

$\pi = \{(\text{on\_ship}, \text{unload}),$   
 $(\text{at\_harbor}, \text{park}),$   
 $(\text{parking1}, \text{deliver})\}$

$Visited = \{\text{on\_ship}, \text{at\_harbor}, \text{parking1}, \text{gate1}\}$





# Find-Acyclic-Solution

Find-Acyclic-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

    if  $Applicable(s) = \emptyset$  then return failure

    nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

    if  $has-loops(\pi, a, Frontier)$  then return failure

return  $\pi$

Keep track of unexpanded states, as in A\*

Add all states  $s$  such that  $\pi(s)$  isn't already defined

*Check for cycles:*

- Does  $\gamma(s, a)$  include a state  $s'$  that is a  $\pi$ -ancestor of  $s$ ?
  - for each  $s' \in \gamma(s, a) \cap Dom(\pi)$ , is  $s \in \hat{\gamma}(s', \pi)$ ?

# Find-Acyclic-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

    if  $Applicable(s) = \emptyset$  then return failure

    nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

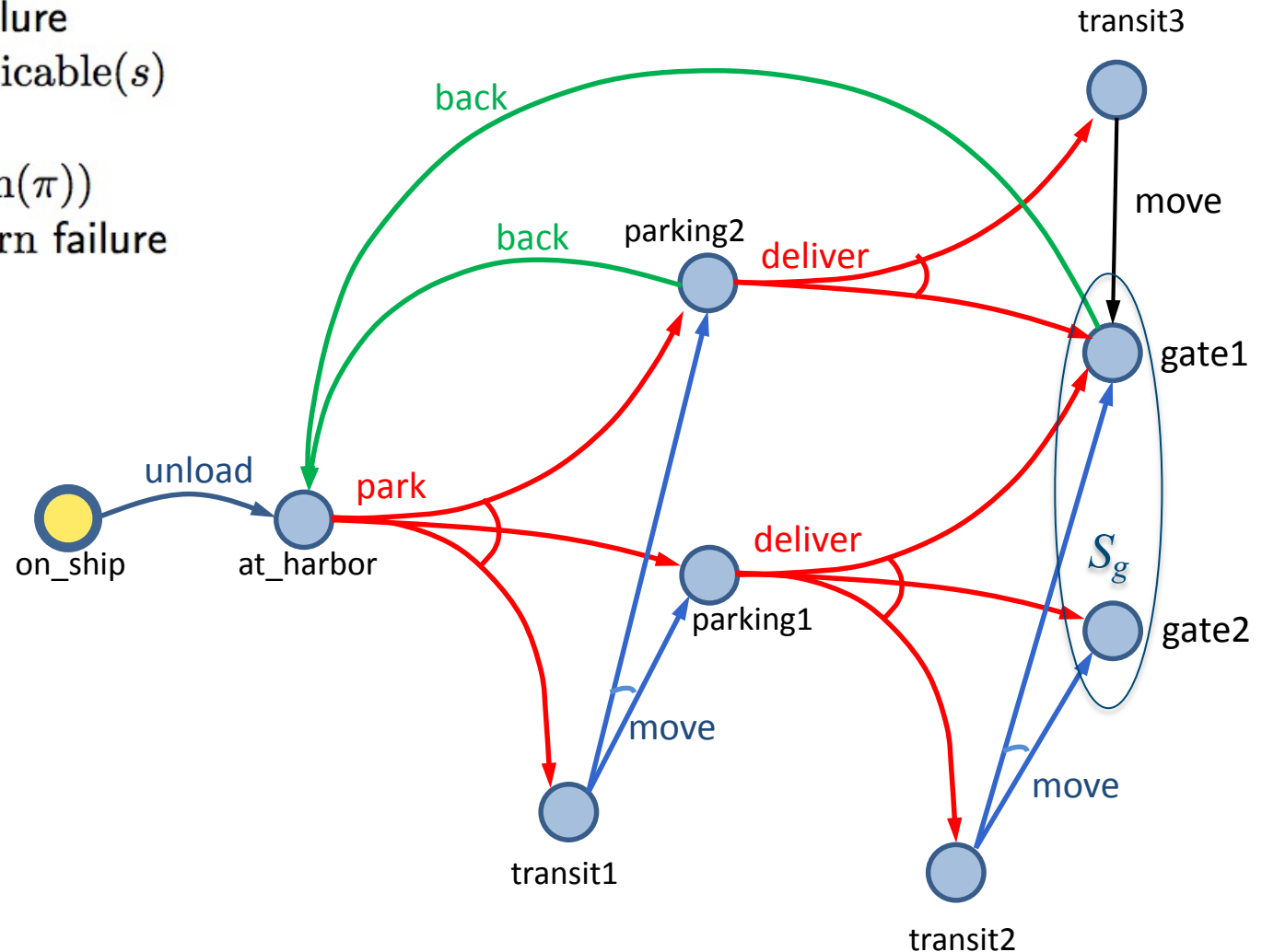
    if  $has-loops(\pi, a, Frontier)$  then return failure

return  $\pi$

$Frontier = \{on\_ship\}$

$\pi = \{\}$

# Example



## Find-Acyclic-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

    if  $Applicable(s) = \emptyset$  then return failure

    nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

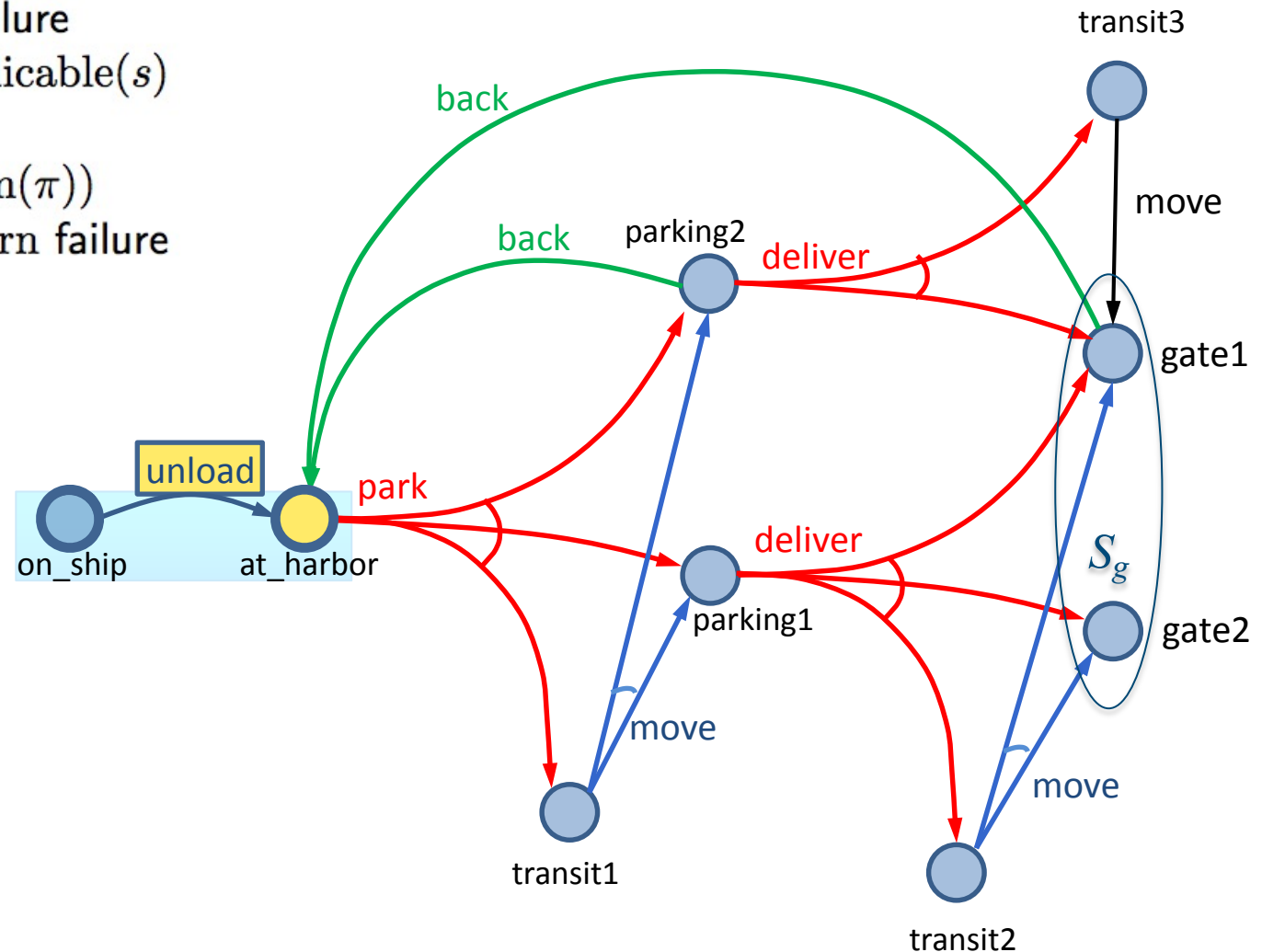
    if  $has-loops(\pi, a, Frontier)$  then return failure

return  $\pi$

$Frontier = \{at\_harbor\}$

$\pi = \{(on\_ship, unload)\}$

## Example



# Find-Acyclic-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

    if  $Applicable(s) = \emptyset$  then return failure

    nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

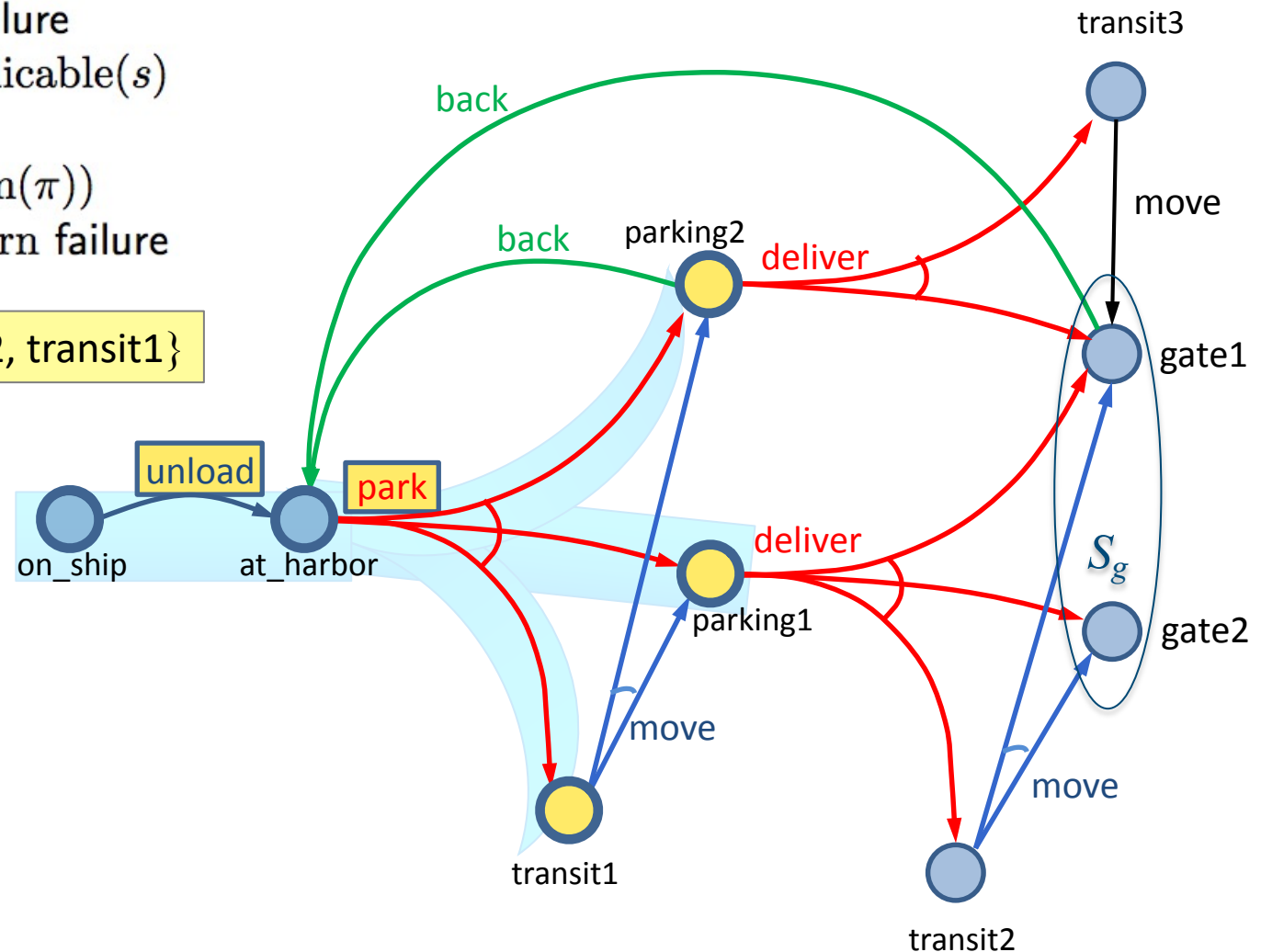
    if  $has-loops(\pi, a, Frontier)$  then return failure

return  $\pi$

$Frontier = \{parking1, parking2, transit1\}$

$\pi = \{(on\_ship, unload), (at\_harbor, park)\}$

# Example



# Find-Acyclic-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

    if  $Applicable(s) = \emptyset$  then return failure

    nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

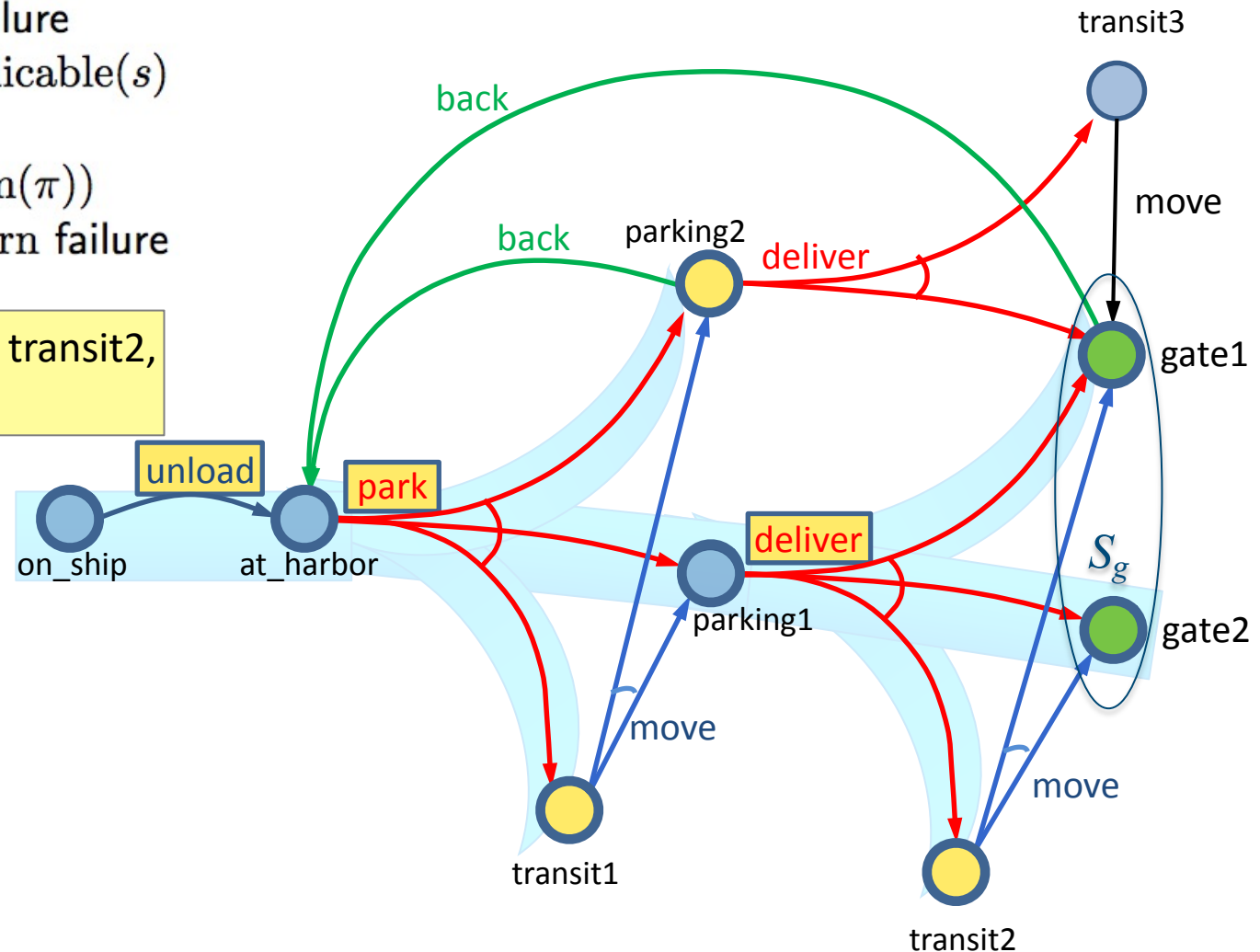
    if  $has-loops(\pi, a, Frontier)$  then return failure

return  $\pi$

$Frontier = \{parking2, transit1, transit2, gate1, gate2\}$

$\pi = \{(on\_ship, unload), (at\_harbor, park), (parking1, deliver)\}$

# Example





## Find-Acyclic-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

if  $Applicable(s) = \emptyset$  then return failure

nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

if  $has-loops(\pi, a, Frontier)$  then return failure

return  $\pi$

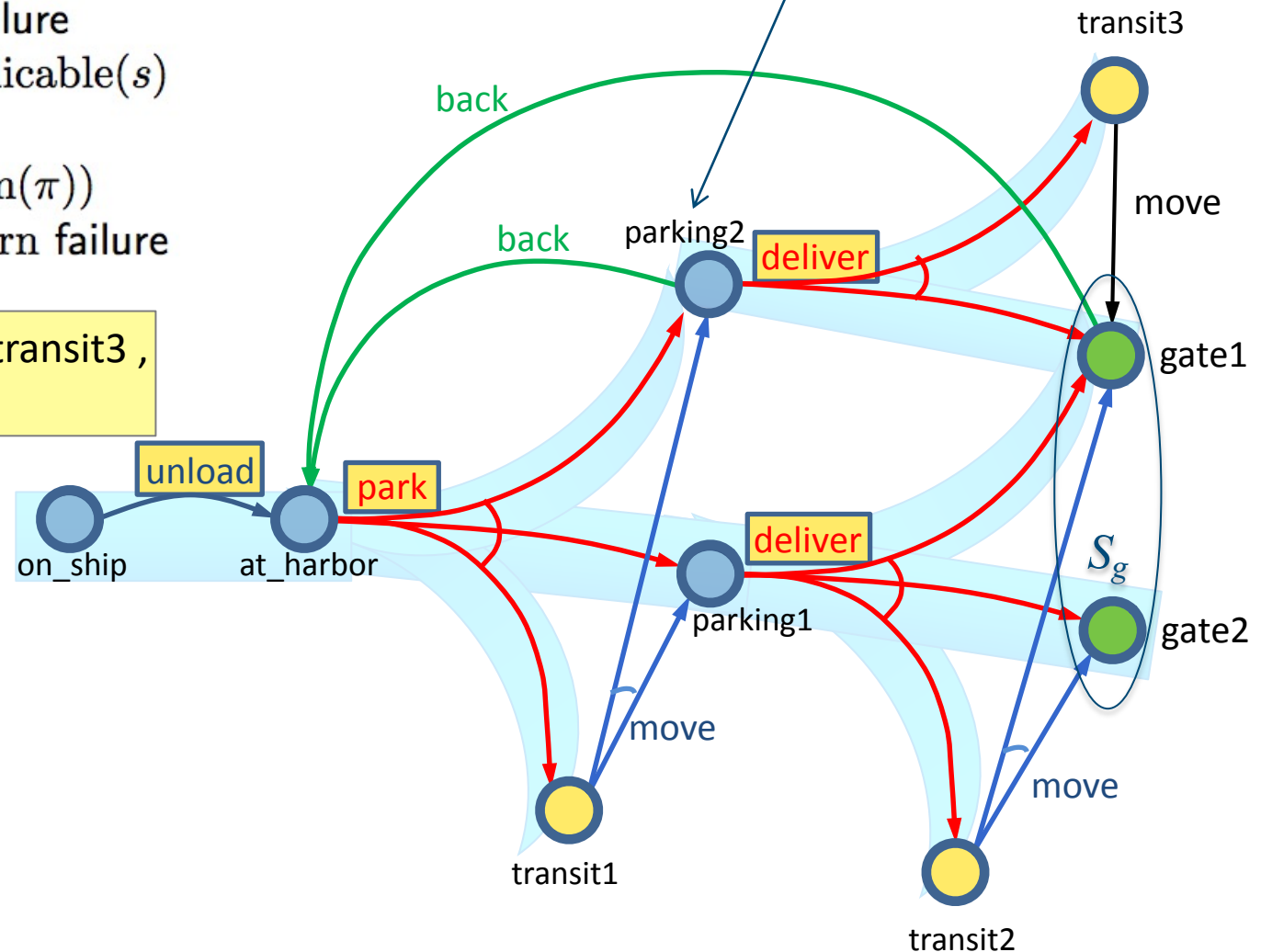
$Frontier = \{transit1, transit2, transit3, gate1, gate2\}$

$\pi = \{(on\_ship, unload), (at\_harbor, park), (parking1, deliver), (parking2, deliver)\}$

## Example

nondeterministically choose back or deliver

- back  $\Rightarrow$  cycle, so return failure
- deliver  $\Rightarrow$  no cycle, so continue





# Find-Acyclic-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

if  $Applicable(s) = \emptyset$  then return failure

nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

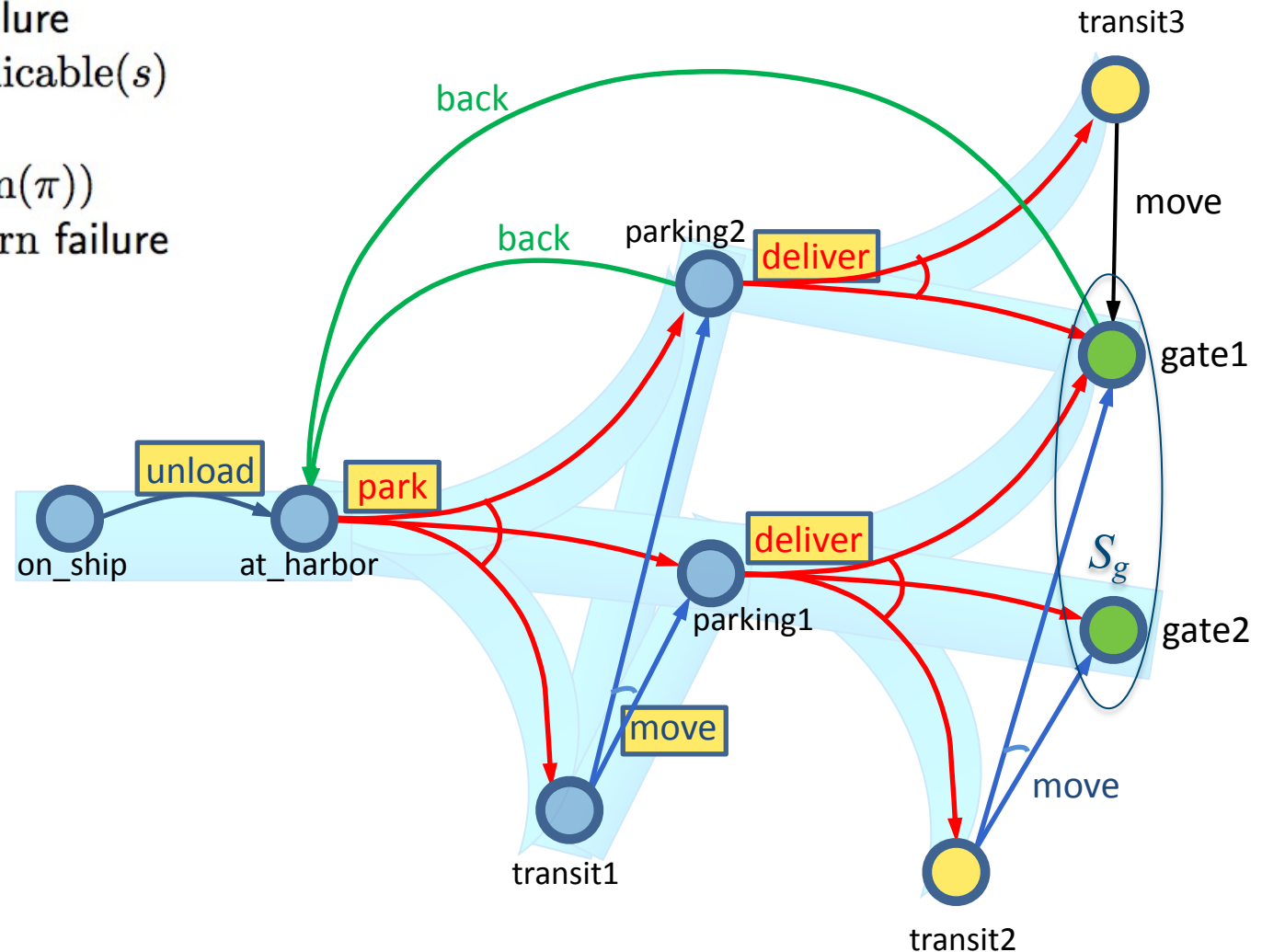
if  $has-loops(\pi, a, Frontier)$  then return failure

return  $\pi$

$Frontier = \{transit2, transit3, gate1, gate2\}$

$\pi = \{(on\_ship, unload), (at\_harbor, park), (parking1, deliver), (parking2, deliver), (transit1, move)\}$

# Example



# Find-Acyclic-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

if  $Applicable(s) = \emptyset$  then return failure

nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

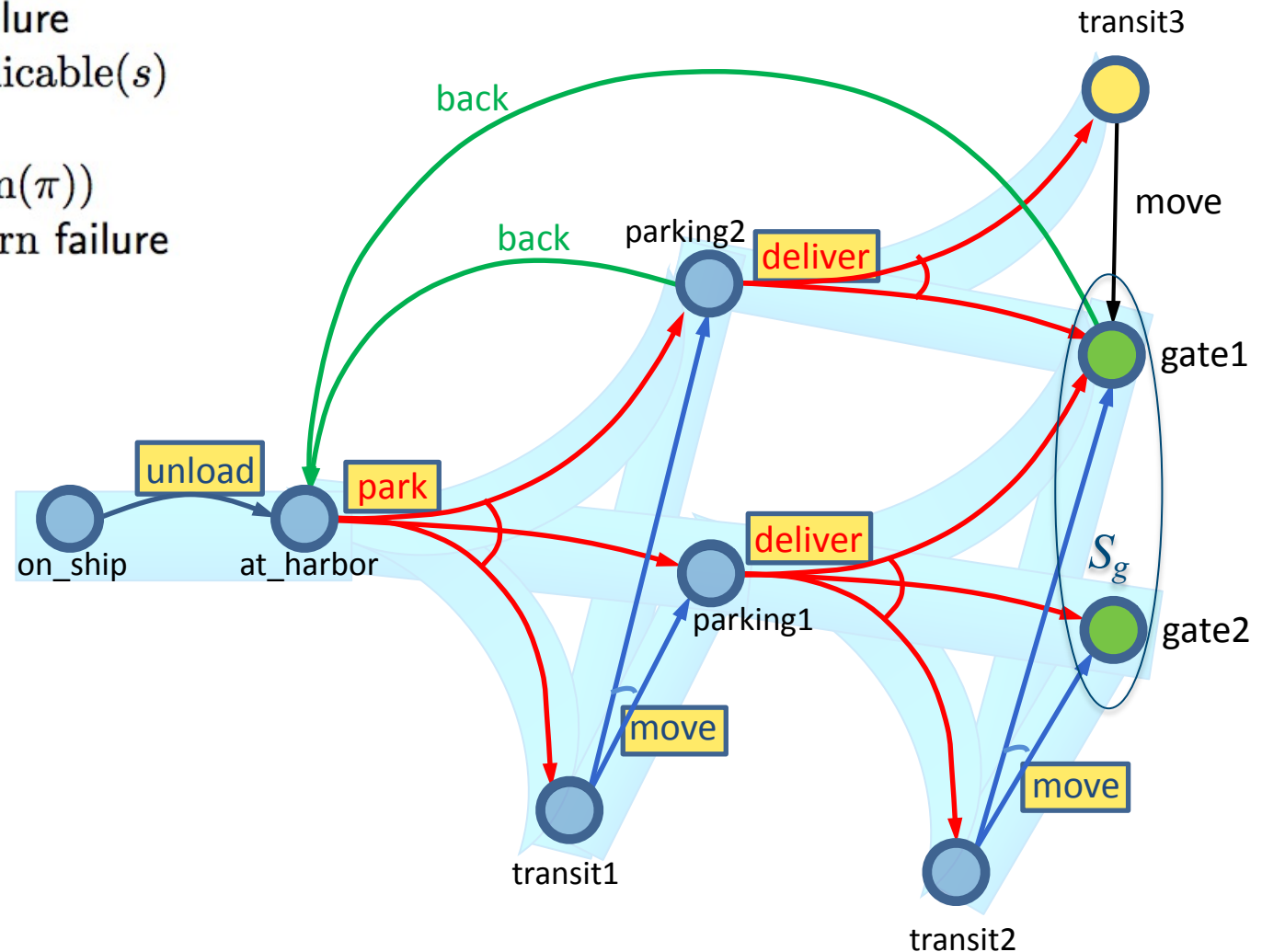
if  $has-loops(\pi, a, Frontier)$  then return failure

return  $\pi$

$Frontier = \{transit3, gate1, gate2\}$

$\pi = \{(on\_ship, unload), (at\_harbor, park), (parking1, deliver), (parking2, deliver), (transit1, move), (transit2, move)\}$

# Example



# Find-Acyclic-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

    if  $Applicable(s) = \emptyset$  then return failure

    nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

    if  $has-loops(\pi, a, Frontier)$  then return failure

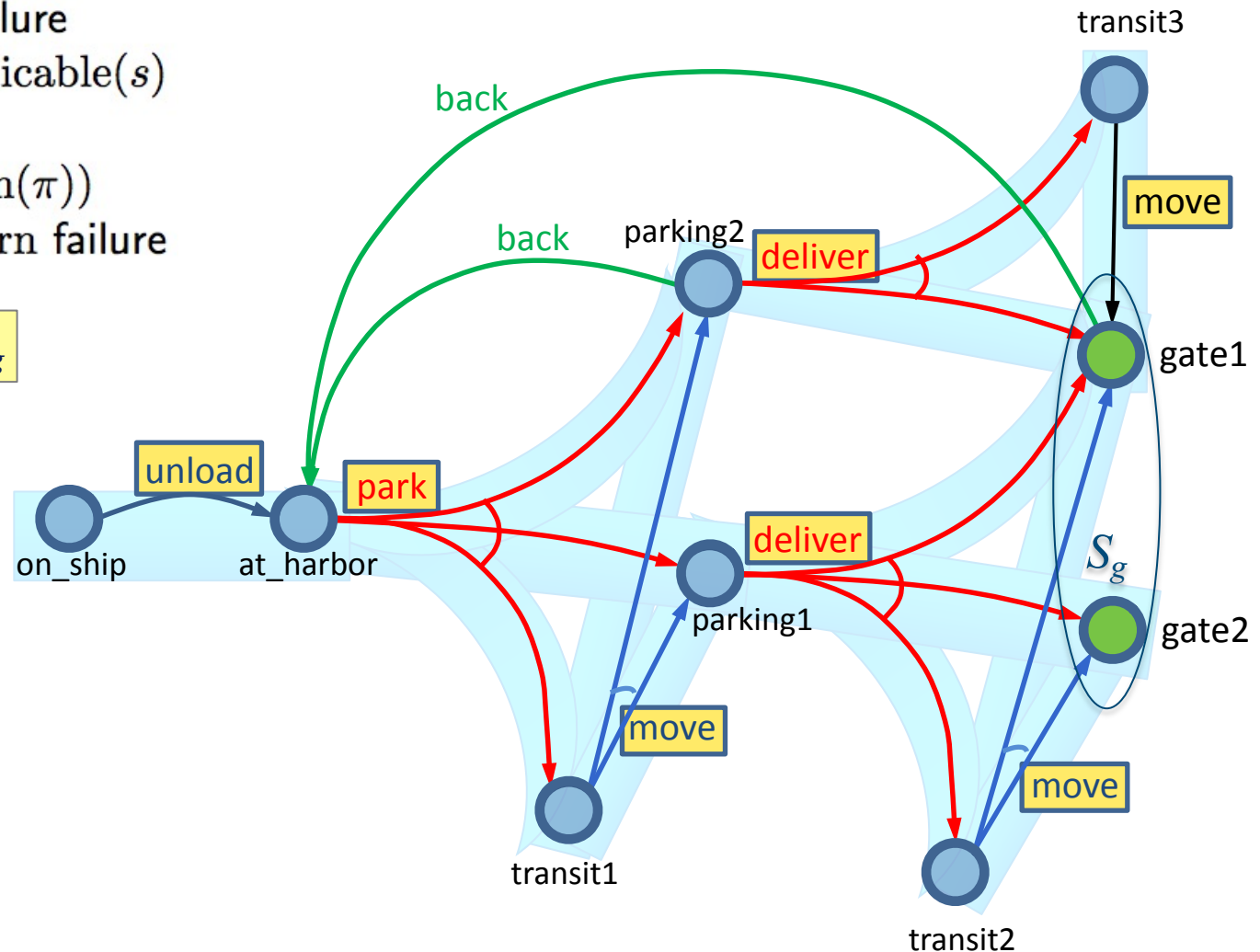
return  $\pi$

$Frontier = \{gate1, gate2\} \subseteq S_g$

Found a solution

$\pi = \{(on\_ship, unload),$   
 $(at\_harbor, park),$   
 $(parking1, deliver),$   
 $(parking2, deliver),$   
 $(transit1, move),$   
 $(transit2, move),$   
 $(transit3, move)\}$

# Example



# Find-Safe-Solution

Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

if  $Applicable(s) = \emptyset$  then return failure

nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

if  $has\text{-unsafe}\text{-loops}(\pi, a, Frontier)$  then return failure

return  $\pi$

Like  
Find-Acyclic-Solution  
except here:

Check for *unsafe* cycles:

- Does  $\gamma(s, a)$  include a state  $s'$  from which  $\pi$  can't take us to the frontier?
  - For each  $s' \in \gamma(s, a) \cap Dom(\pi)$ , is  $\hat{\gamma}(s', \pi) \cap Frontier = \emptyset$ ?
- If so, then  $\pi$  contains a cycle that can't be escaped

# Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

    if  $Applicable(s) = \emptyset$  then return failure

    nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

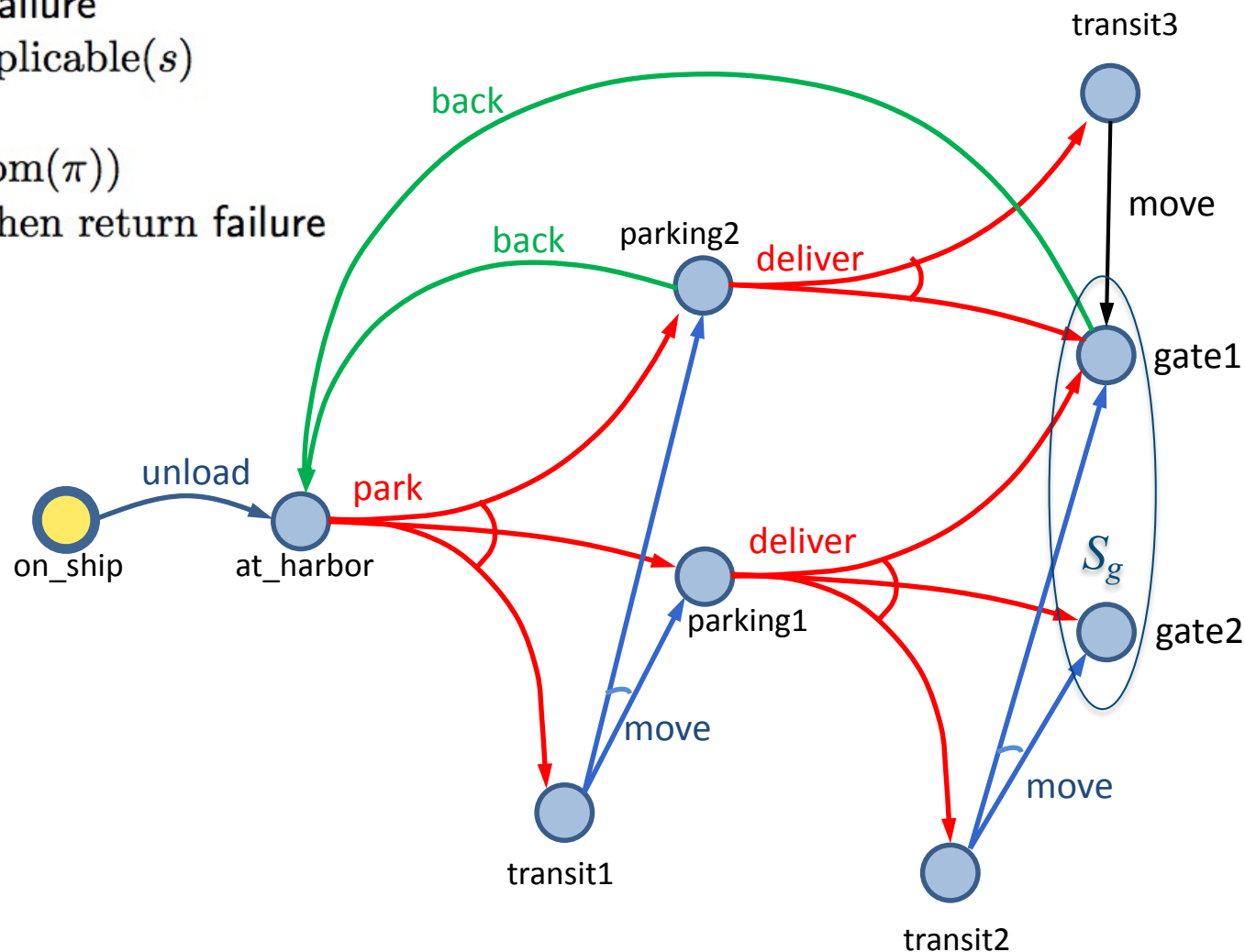
    if  $has\_unsafe\_loops(\pi, a, Frontier)$  then return failure

return  $\pi$

$Frontier = \{on\_ship\}$

$\pi = \{\}$

# Example





# Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

    if  $Applicable(s) = \emptyset$  then return failure

    nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

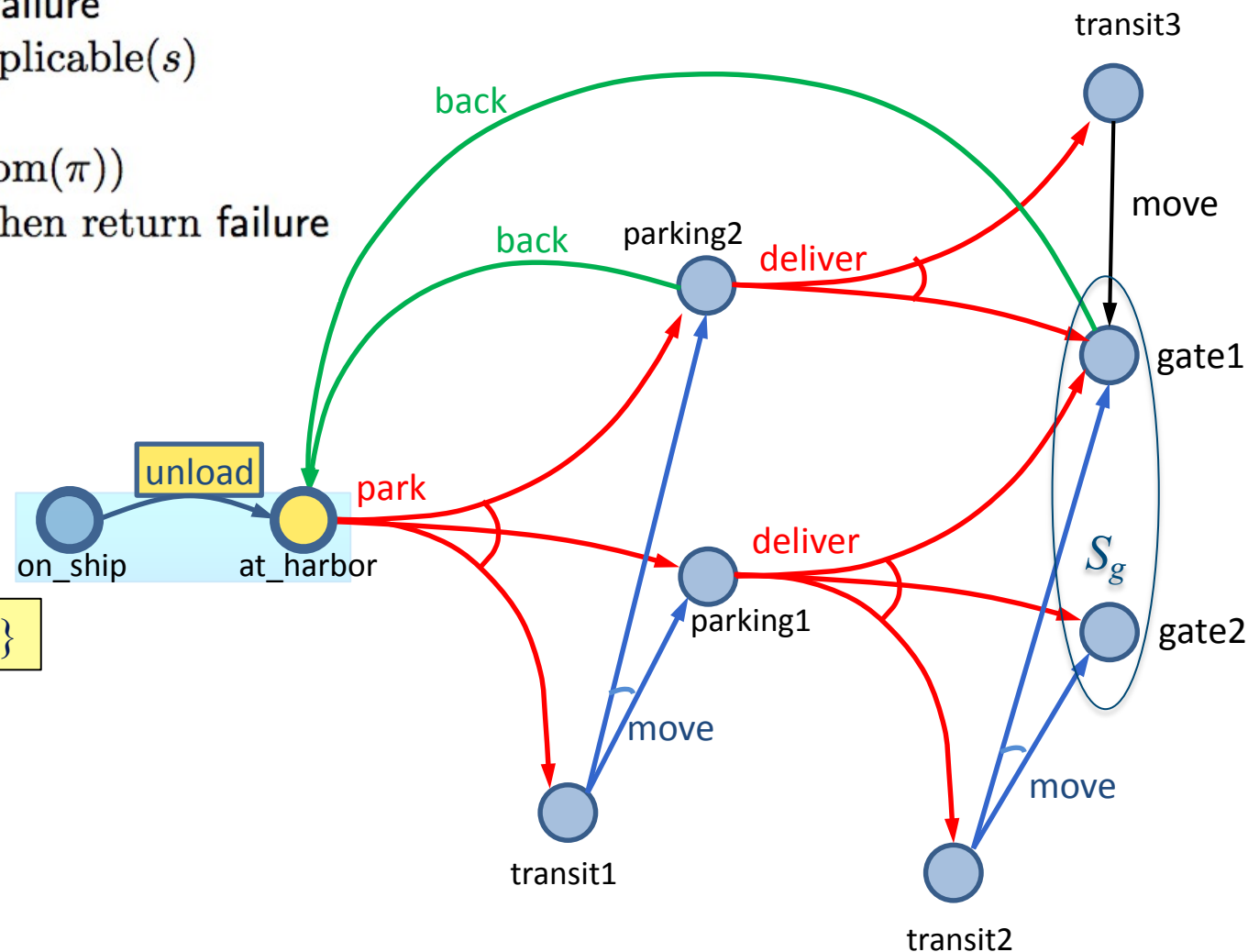
    if  $has\_unsafe\_loops(\pi, a, Frontier)$  then return failure

return  $\pi$

$Frontier = \{at\_harbor\}$

$\pi = \{(on\_ship, unload)\}$

# Example





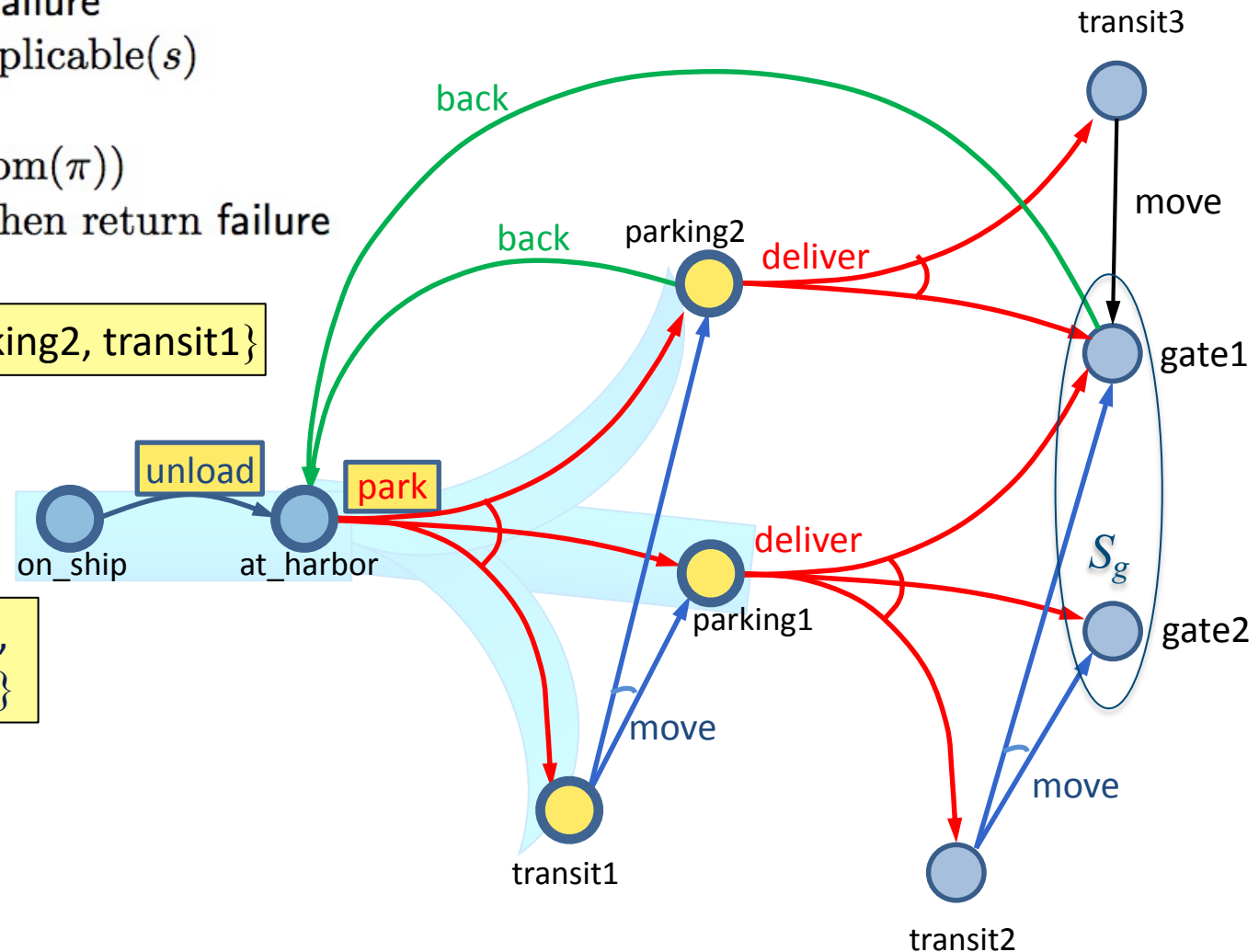
# Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

```
 $\pi \leftarrow \emptyset$   
 $Frontier \leftarrow \{s_0\}$   
for every  $s \in Frontier \setminus S_g$  do  
   $Frontier \leftarrow Frontier \setminus \{s\}$   
  if  $Applicable(s) = \emptyset$  then return failure  
  nondeterministically choose  $a \in Applicable(s)$   
   $\pi \leftarrow \pi \cup (s, a)$   
   $Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$   
  if  $has\_unsafe\_loops(\pi, a, Frontier)$  then return failure  
return  $\pi$ 
```

$Frontier = \{parking1, parking2, transit1\}$

$\pi = \{(on\_ship, unload), (at\_harbor, park)\}$

# Example



# Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

    if  $Applicable(s) = \emptyset$  then return failure

    nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

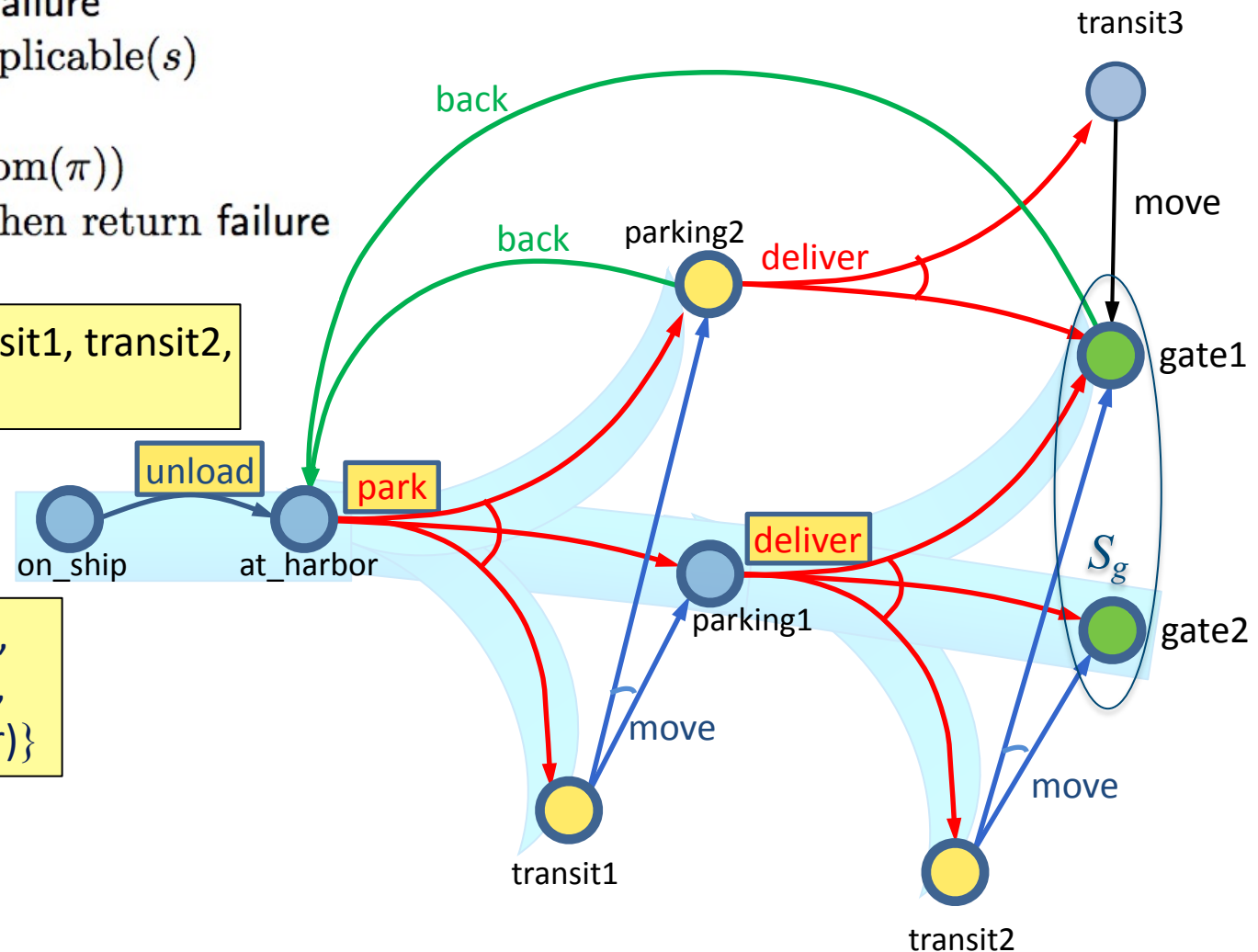
    if  $has\_unsafe\_loops(\pi, a, Frontier)$  then return failure

return  $\pi$

$Frontier = \{parking2, transit1, transit2, gate1, gate2\}$

$\pi = \{(on\_ship, unload), (at\_harbor, park), (parking1, deliver)\}$

# Example



# Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

if  $Applicable(s) = \emptyset$  then return failure

nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

if  $has\_unsafe\_loops(\pi, a, Frontier)$  then return failure

return  $\pi$

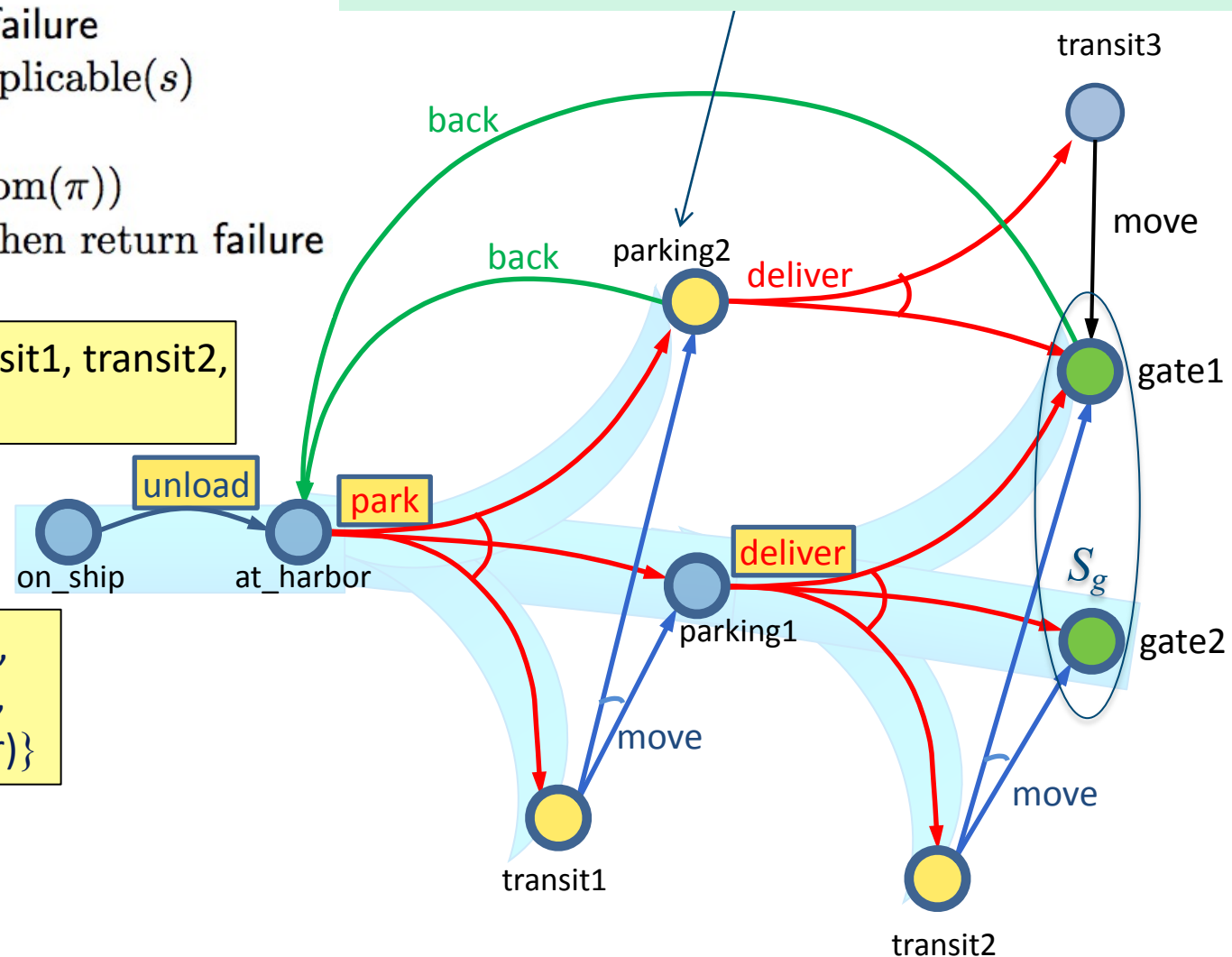
$Frontier = \{parking2, transit1, transit2, gate1, gate2\}$

$\pi = \{(on\_ship, unload), (at\_harbor, park), (parking1, deliver)\}$

# Example

Nondeterministically choose either back or deliver

- back is OK because cycle is escapable



# Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

    if  $Applicable(s) = \emptyset$  then return failure

    nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$

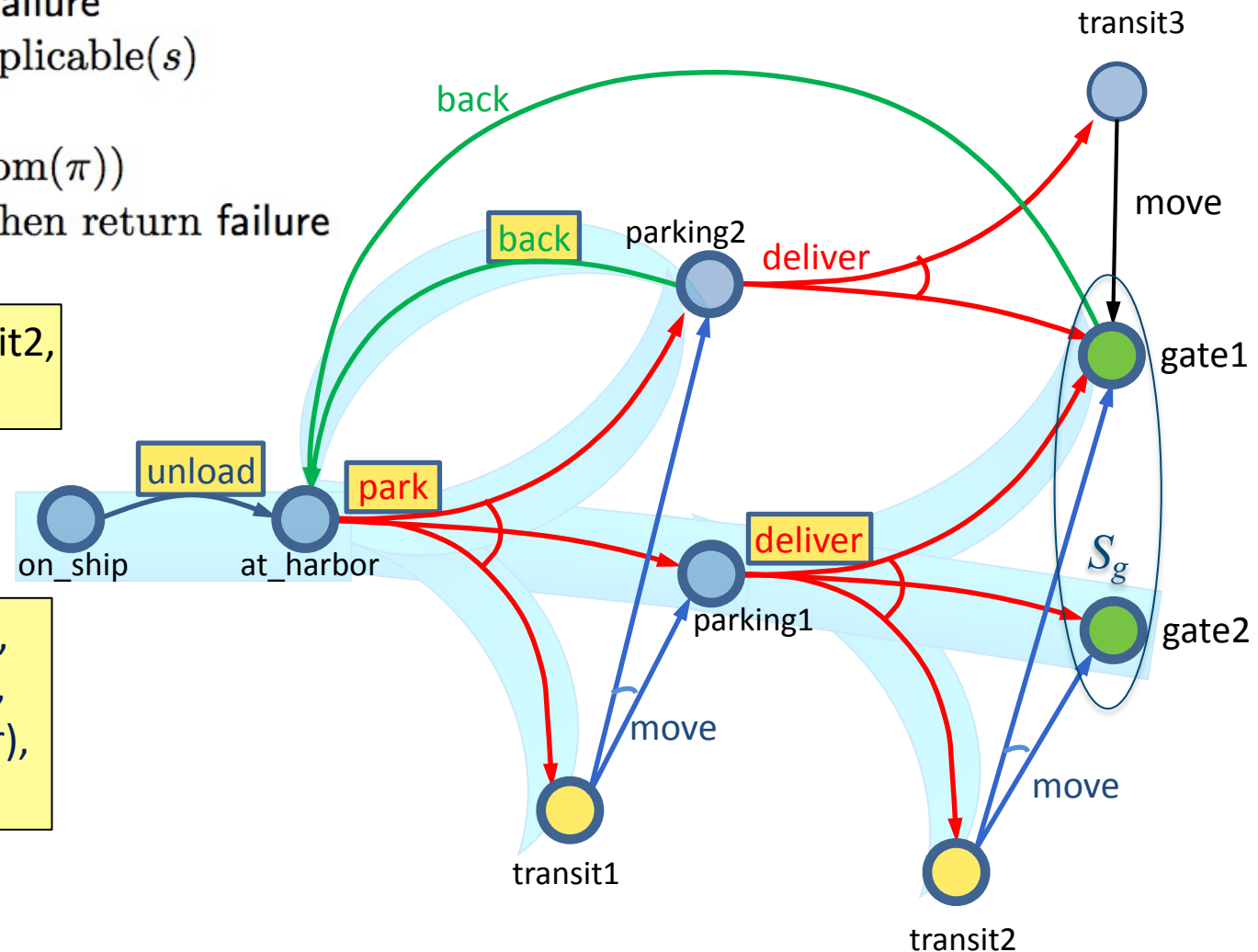
    if  $has\_unsafe\_loops(\pi, a, Frontier)$  then return failure

return  $\pi$

$Frontier = \{transit1, transit2, gate1, gate2\}$

$\pi = \{(on\_ship, unload), (at\_harbor, park), (parking1, deliver), (parking2, back)\}$

# Example



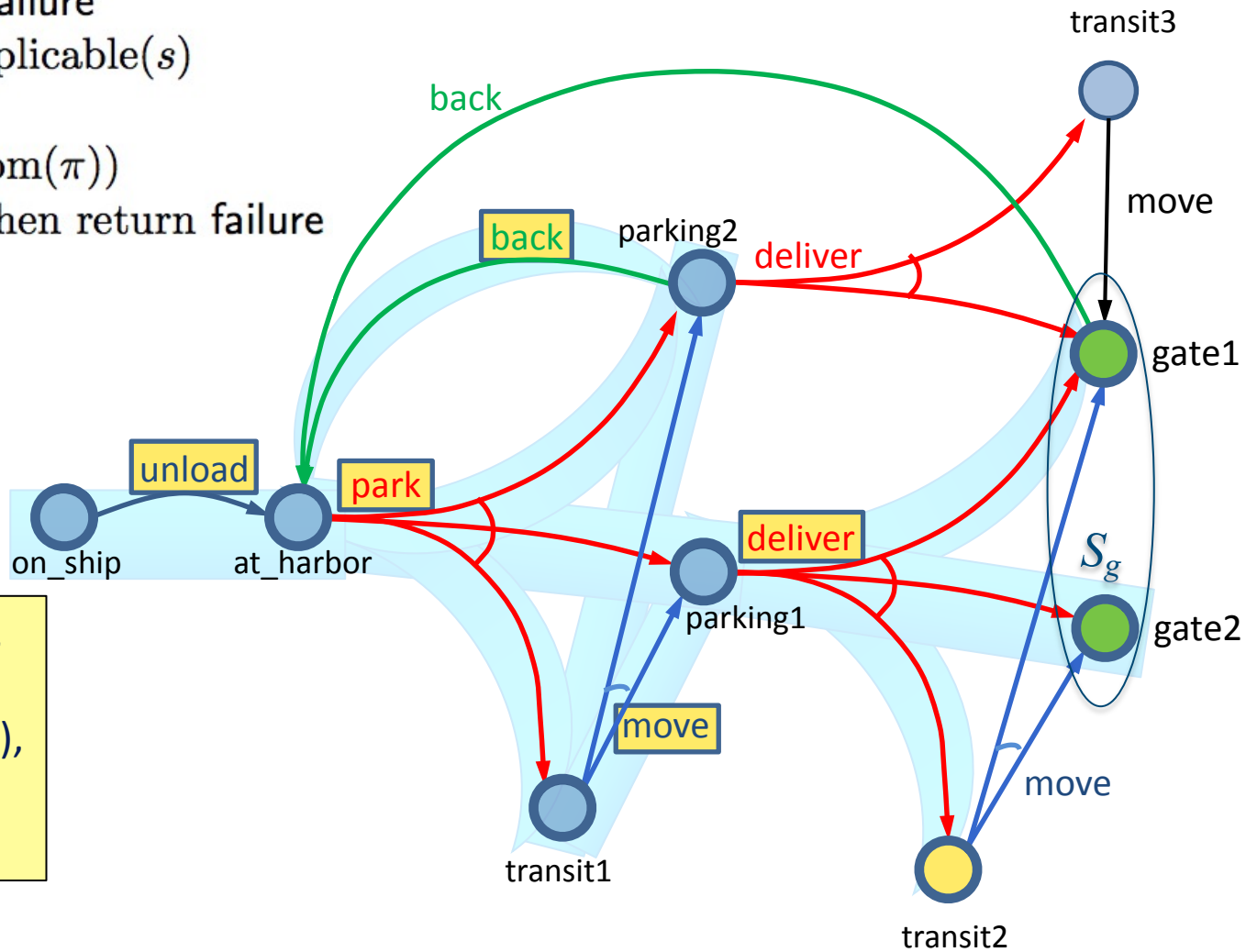
# Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

```
 $\pi \leftarrow \emptyset$   
 $Frontier \leftarrow \{s_0\}$   
for every  $s \in Frontier \setminus S_g$  do  
   $Frontier \leftarrow Frontier \setminus \{s\}$   
  if  $Applicable(s) = \emptyset$  then return failure  
  nondeterministically choose  $a \in Applicable(s)$   
   $\pi \leftarrow \pi \cup (s, a)$   
   $Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$   
  if  $has\_unsafe\_loops(\pi, a, Frontier)$  then return failure  
return  $\pi$ 
```

$Frontier = \{transit2, gate1, gate2\}$

$\pi = \{(on\_ship, unload), (at\_harbor, park), (parking1, deliver), (parking2, back), (transit1, move)\}$

# Example





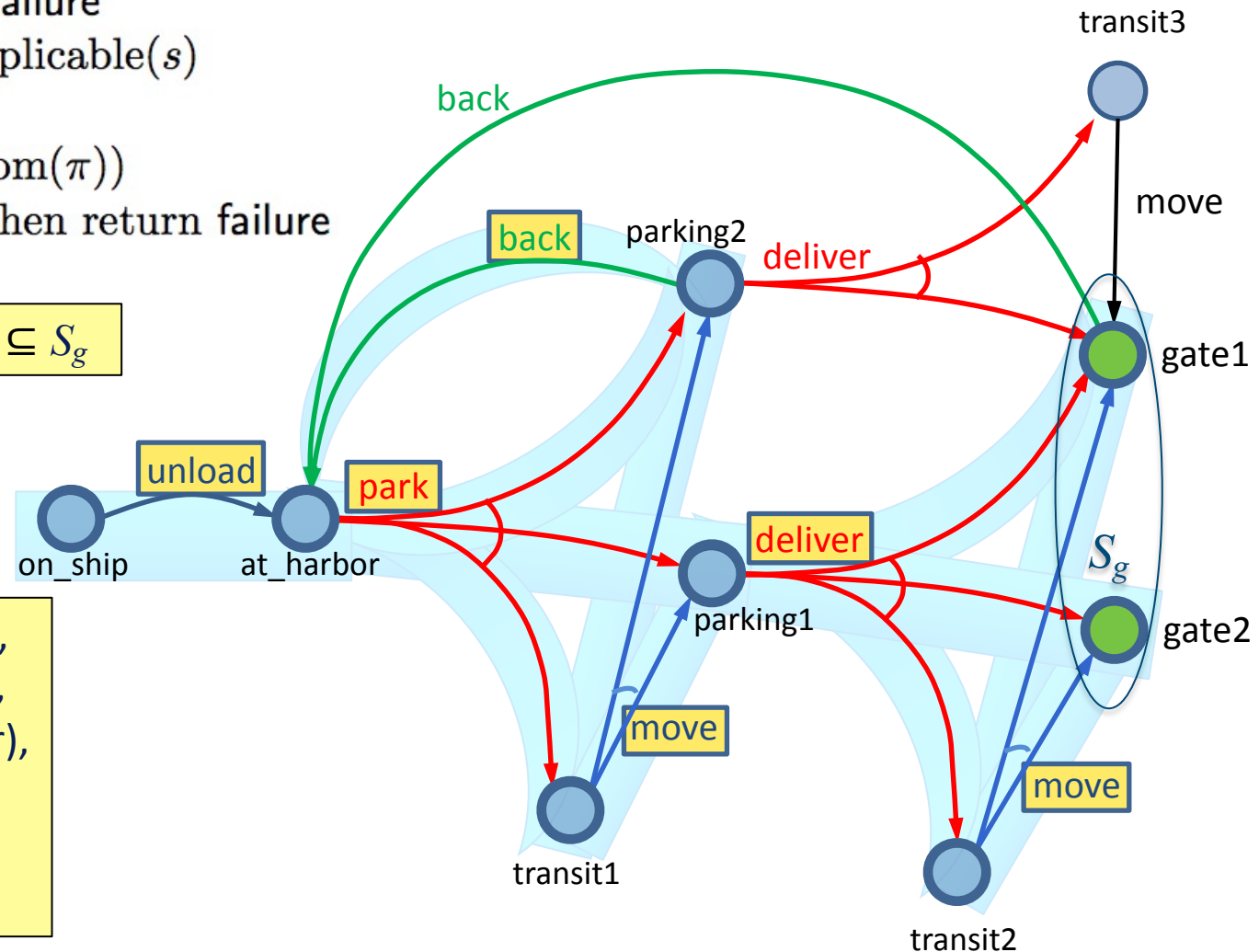
# Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

```
 $\pi \leftarrow \emptyset$   
 $Frontier \leftarrow \{s_0\}$   
for every  $s \in Frontier \setminus S_g$  do  
   $Frontier \leftarrow Frontier \setminus \{s\}$   
  if  $Applicable(s) = \emptyset$  then return failure  
  nondeterministically choose  $a \in Applicable(s)$   
   $\pi \leftarrow \pi \cup (s, a)$   
   $Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$   
  if  $has\_unsafe\_loops(\pi, a, Frontier)$  then return failure  
return  $\pi$ 
```

$Frontier = \{gate1, gate2\} \subseteq S_g$

$\pi = \{(on\_ship, unload),$   
 $(at\_harbor, park),$   
 $(parking1, deliver),$   
 $(parking2, back),$   
 $(transit1, move),$   
 $(transit2, move)\}$

# Example





# Guided-Find-Safe-Solution

- Motivation: much easier to find solutions if they don't have to be safe
  - ▶ Find-Safe-Solution needs plans for all possible outcomes of actions
  - ▶ Find-Solution only needs a plan for one of them
- Idea:
  - ▶ loop
    - Find a (possibly unsafe) solution  $\pi$
    - For each each leaf node of  $\pi$ 
      - ▶ If the leaf node isn't a goal,
        - find a (possibly unsafe) solution and incorporate it into  $\pi$

# Guided-Find-Safe-Solution

Guided-Find-Safe-Solution  $(\Sigma, s_0, S_g)$

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

loop

$Q \leftarrow leaves(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select arbitrarily  $s \in Q$

$\pi' \leftarrow \text{Find-Solution}(\Sigma, s, S_g)$

if  $\pi' \neq \text{failure}$  then do

$\pi \leftarrow \pi \cup \{(s, a) \in \pi' \mid s \notin \text{Dom}(\pi)\}$

else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make  $a$  not applicable in  $s'$

$\pi$  is a solution. Return the part that's reachable from  $s_0$ .

Choose any leaf  $s$  that isn't a goal.  
Find a (possibly unsafe) solution  $\pi'$  for  $s$ .

For each  $(s, a)$  in  $\pi'$ , add to  $\pi$  unless  $\pi$  already has an action at  $s$

$s$  is unsolvable. For each  $(s', a)$  that can produce  $s$ , modify  $\pi$  and  $\Sigma$  so we'll never use  $a$  at  $s'$

# Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

loop

$Q \leftarrow leaves(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select arbitrarily  $s \in Q$

$\pi' \leftarrow Find\text{-}Solution(\Sigma, s, S_g)$

if  $\pi' \neq failure$  then do

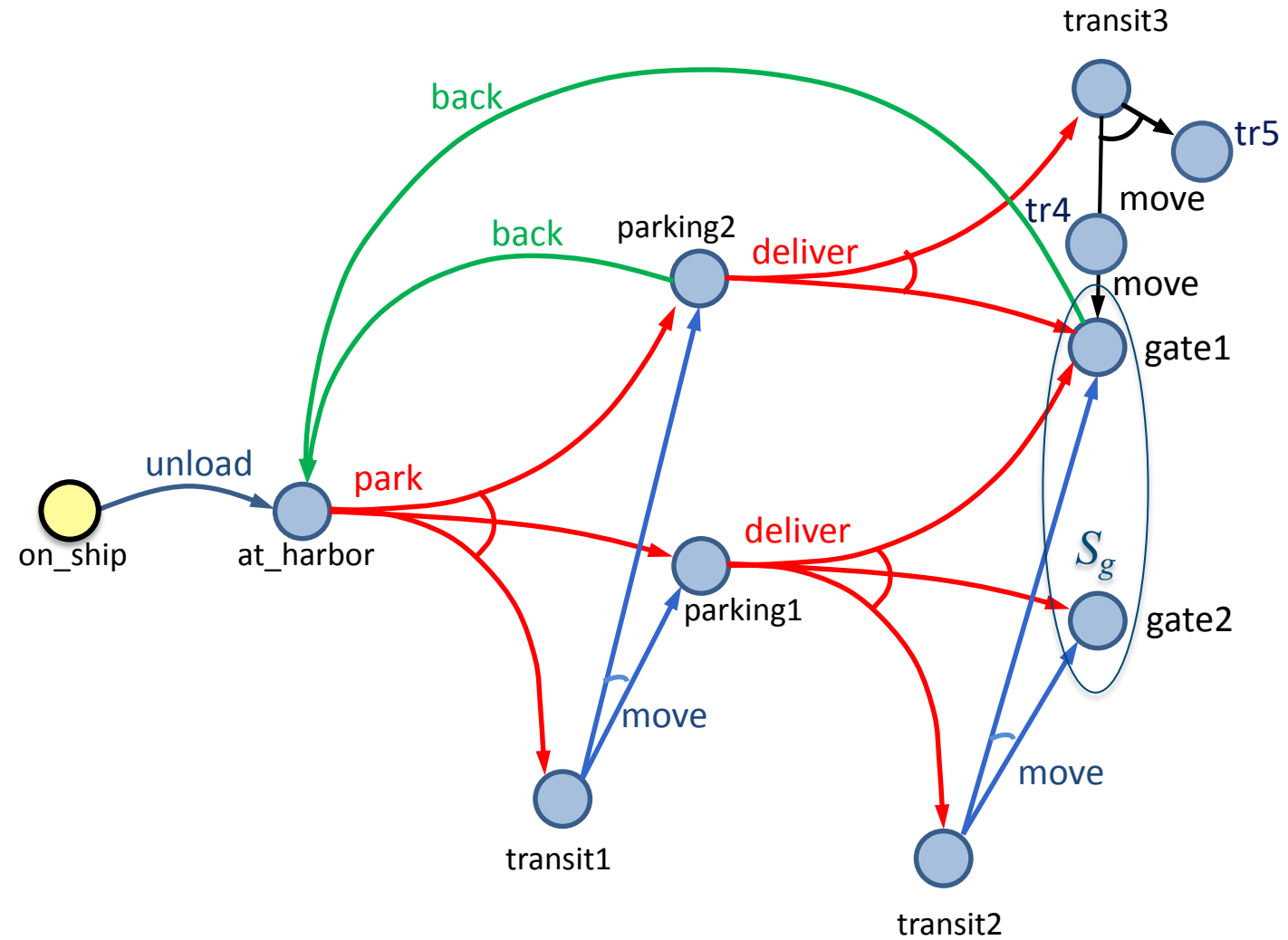
$\pi \leftarrow \pi \cup \{(s, a) \in \pi' \mid s \notin Dom(\pi)\}$

else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make  $a$  not applicable in  $s'$

# Example



## Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

loop

$Q \leftarrow leaves(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select arbitrarily  $s \in Q$

$\pi' \leftarrow Find\text{-}Solution(\Sigma, s, S_g)$

if  $\pi' \neq failure$  then do

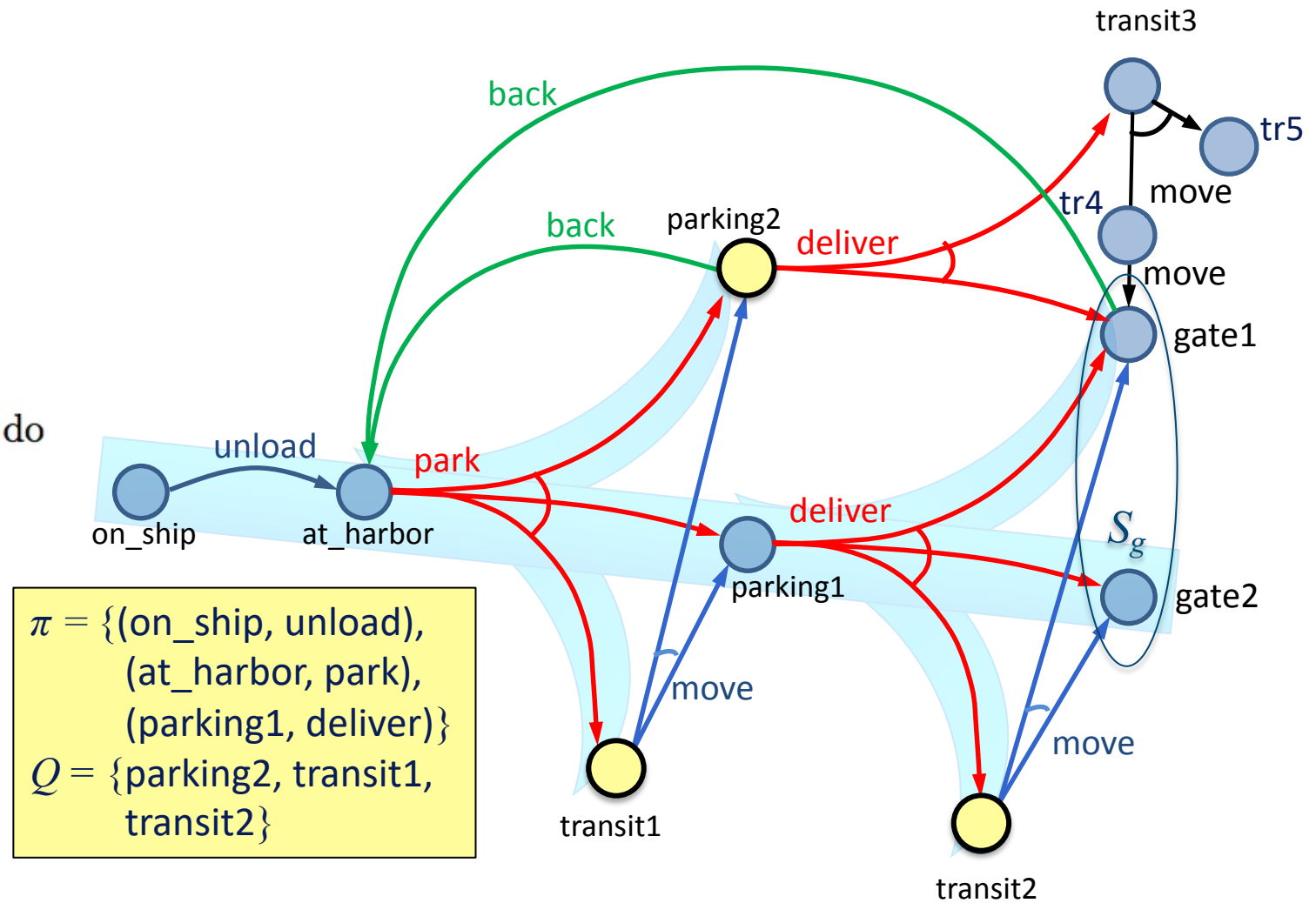
$\pi \leftarrow \pi \cup \{(s, a) \in \pi' \mid s \notin Dom(\pi)\}$

else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make  $a$  not applicable in  $s'$

## Example



## Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

loop

$Q \leftarrow leaves(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select arbitrarily  $s \in Q$

$\pi' \leftarrow Find\text{-}Solution(\Sigma, s, S_g)$

if  $\pi' \neq failure$  then do

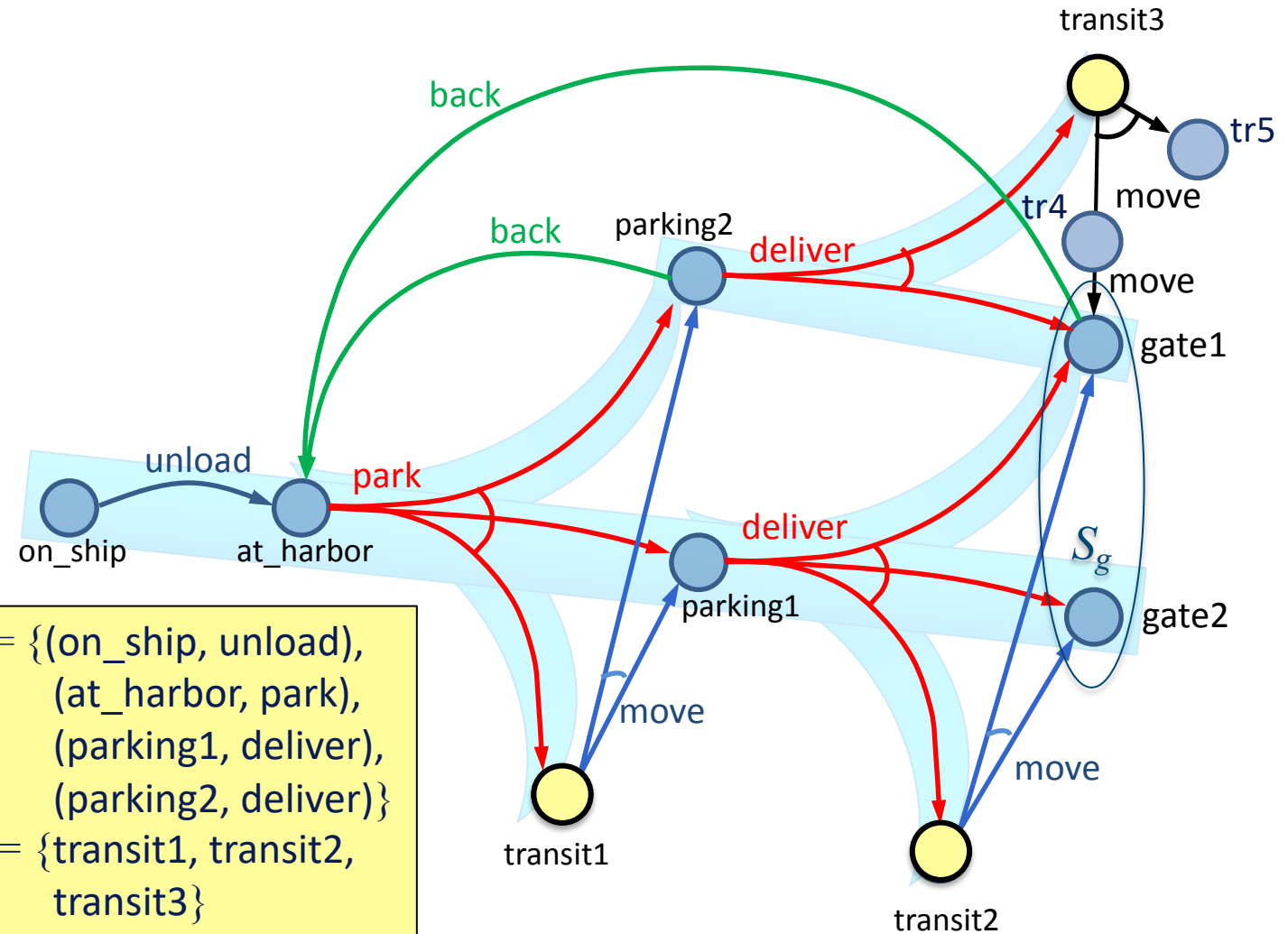
$\pi \leftarrow \pi \cup \{(s, a) \in \pi' \mid s \notin Dom(\pi)\}$

else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make  $a$  not applicable in  $s'$

## Example





## Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

loop

$Q \leftarrow leaves(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select arbitrarily  $s \in Q$

$\pi' \leftarrow Find-Solution(\Sigma, s, S_g)$

if  $\pi' \neq failure$  then do

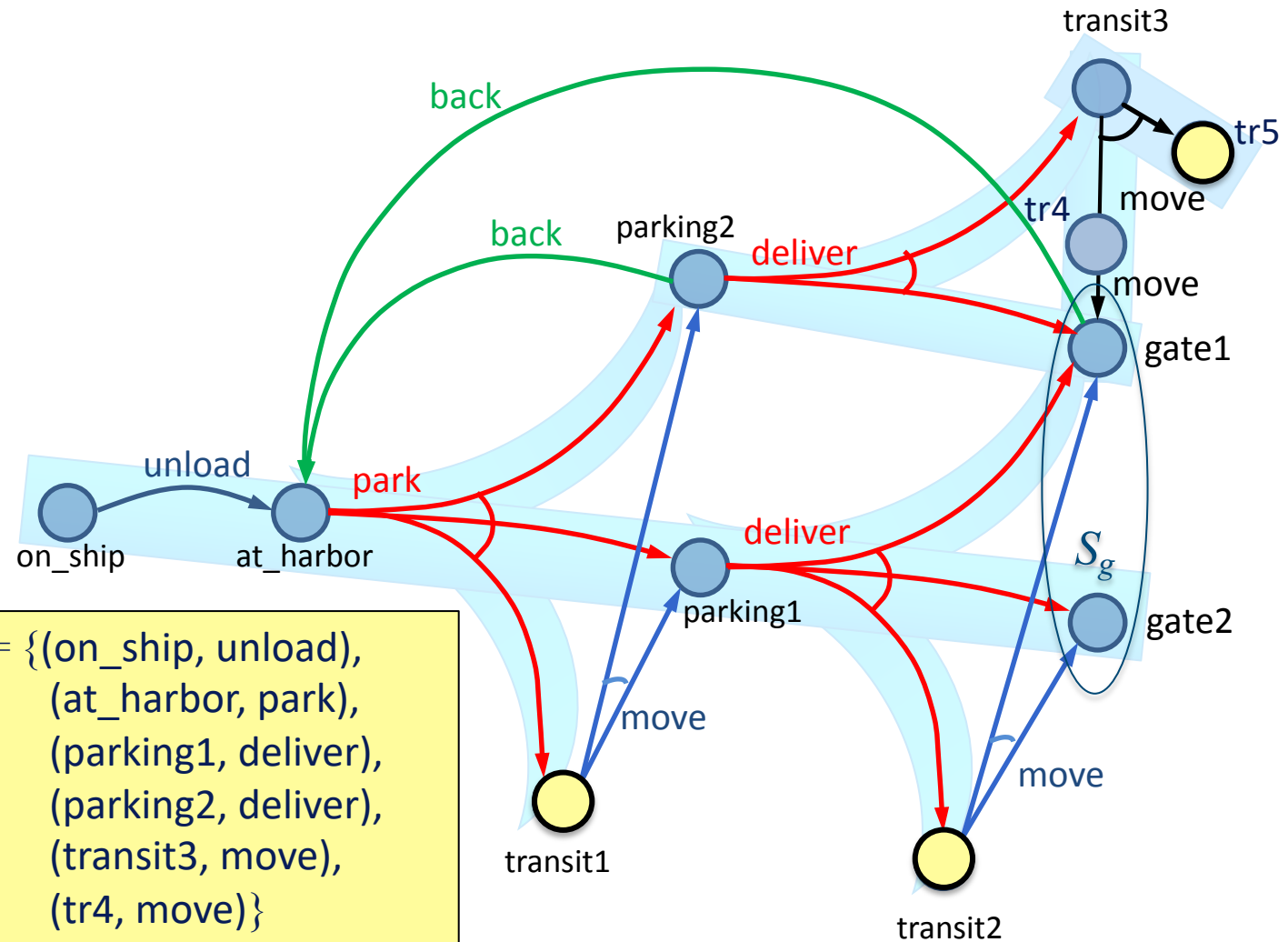
$\pi \leftarrow \pi \cup \{(s, a) \in \pi' \mid s \notin Dom(\pi)\}$

else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make  $a$  not applicable in  $s'$

## Example





## Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

loop

$Q \leftarrow leaves(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select arbitrarily  $s \in Q$

$\pi' \leftarrow Find\text{-}Solution(\Sigma, s, S_g)$

if  $\pi' \neq failure$  then do

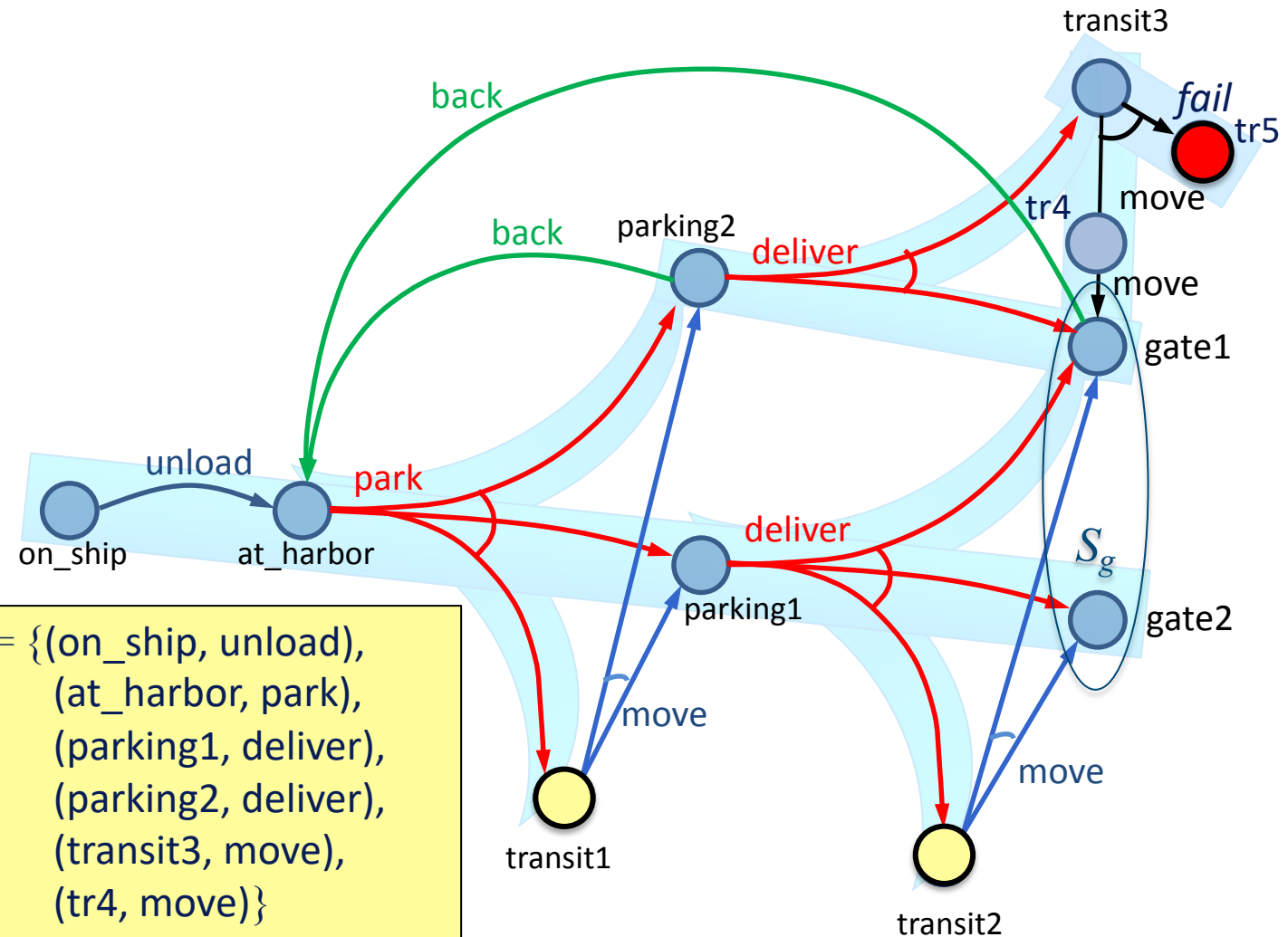
$\pi \leftarrow \pi \cup \{(s, a) \in \pi' \mid s \notin Dom(\pi)\}$

else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make  $a$  not applicable in  $s'$

## Example



$\pi = \{(on\_ship, unload),$   
 $(at\_harbor, park),$   
 $(parking1, deliver),$   
 $(parking2, deliver),$   
 $(transit3, move),$   
 $(tr4, move)\}$   
 $Q = \{transit1, transit2, tr5\}$

## Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

loop

$Q \leftarrow leaves(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select arbitrarily  $s \in Q$

$\pi' \leftarrow Find-Solution(\Sigma, s, S_g)$

if  $\pi' \neq failure$  then do

$\pi \leftarrow \pi \cup \{(s, a) \in \pi' \mid s \notin Dom(\pi)\}$

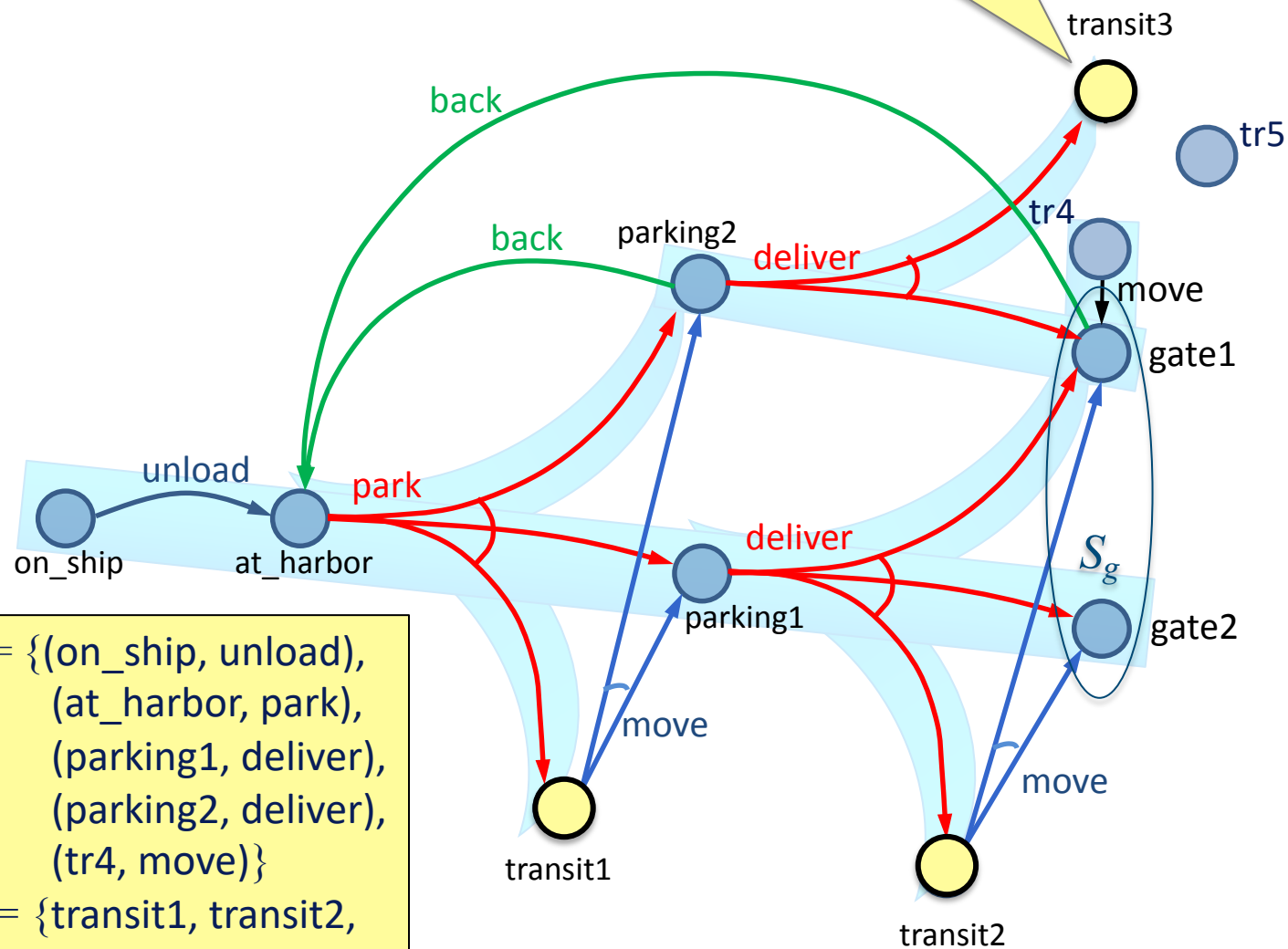
else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make  $a$  not applicable in  $s'$

## Example

Modify  $\Sigma_d$  to make  
move inapplicable at  
transit3



$\pi = \{(on\_ship, unload),$   
 $(at\_harbor, park),$   
 $(parking1, deliver),$   
 $(parking2, deliver),$   
 $(tr4, move)\}$

$Q = \{transit1, transit2,$   
 $transit3\}$

## Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

loop

$Q \leftarrow leaves(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select arbitrarily  $s \in Q$

$\pi' \leftarrow Find-Solution(\Sigma, s, S_g)$

if  $\pi' \neq failure$  then do

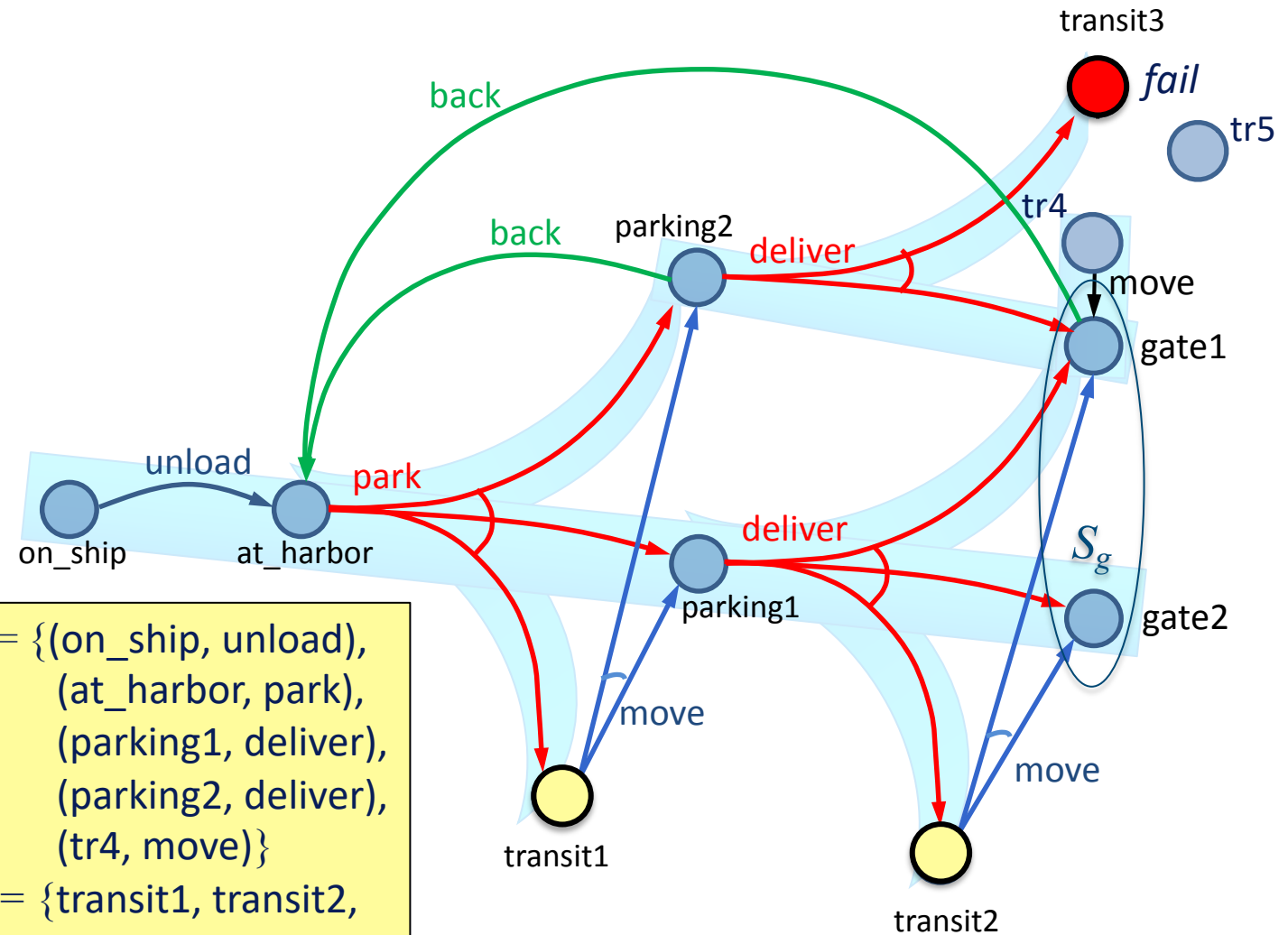
$\pi \leftarrow \pi \cup \{(s, a) \in \pi' \mid s \notin Dom(\pi)\}$

else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make  $a$  not applicable in  $s'$

## Example



## Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

loop

$Q \leftarrow leaves(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select arbitrarily  $s \in Q$

$\pi' \leftarrow \text{Find-Solution}(\Sigma, s, S_g)$

if  $\pi' \neq \text{failure}$  then do

$\pi \leftarrow \pi \cup \{(s, a) \in \pi' \mid s \notin \text{Dom}(\pi)\}$

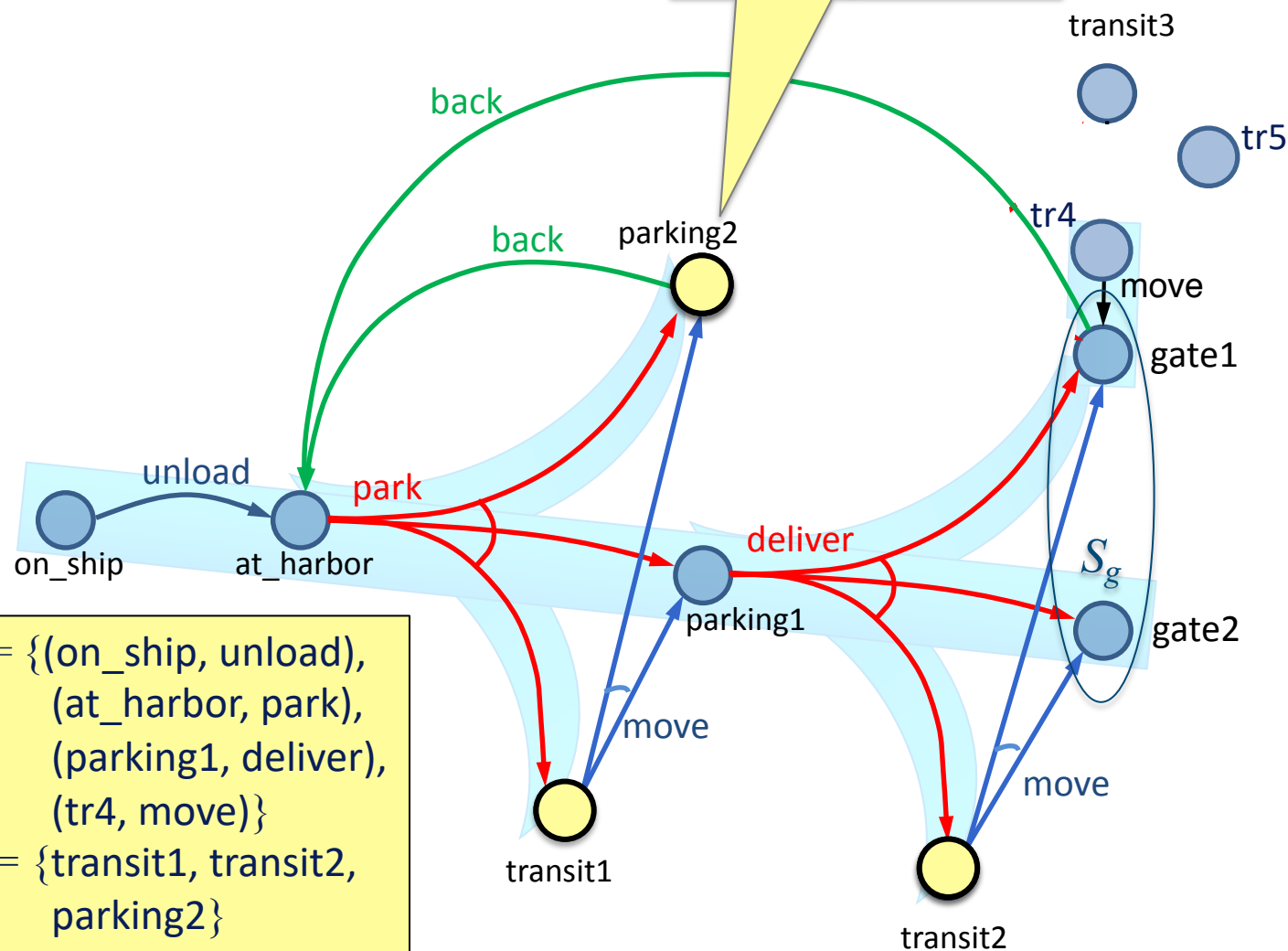
else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make  $a$  not applicable in  $s'$

## Example

Modify  $\Sigma_d$  to make deliver inapplicable at parking2





## Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

loop

$Q \leftarrow leaves(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select arbitrarily  $s \in Q$

$\pi' \leftarrow Find-Solution(\Sigma, s, S_g)$

if  $\pi' \neq failure$  then do

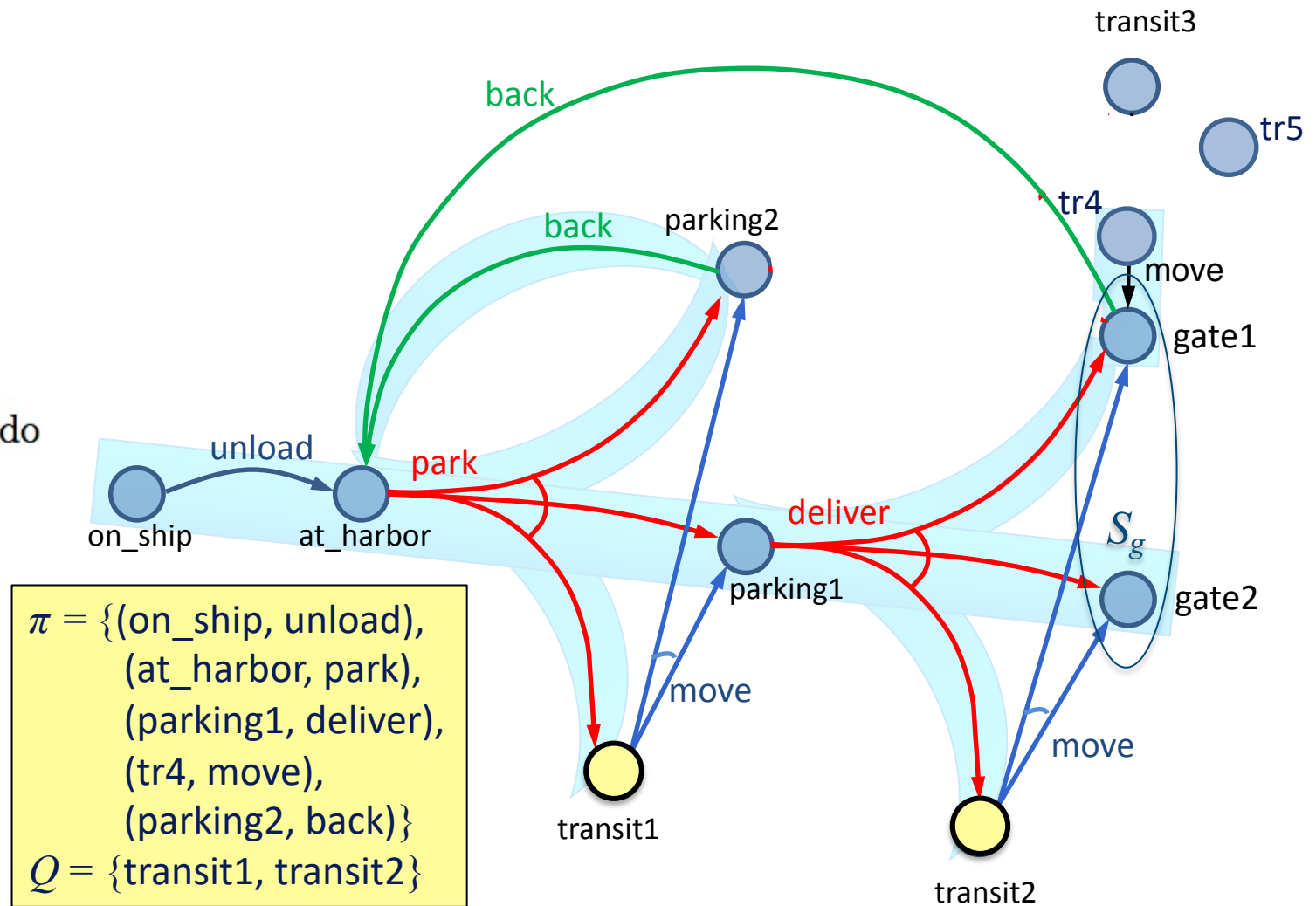
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## Example



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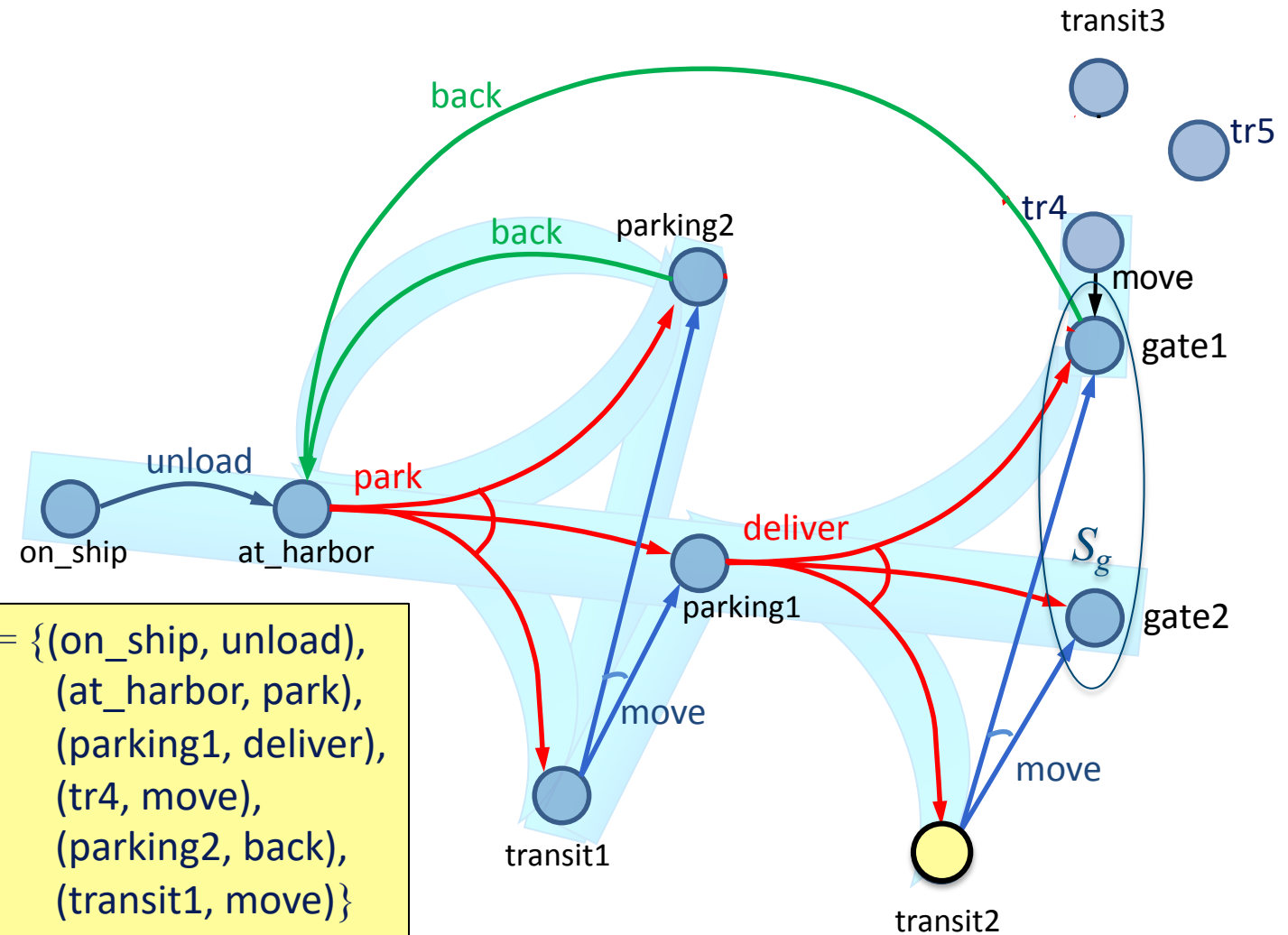
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## Example



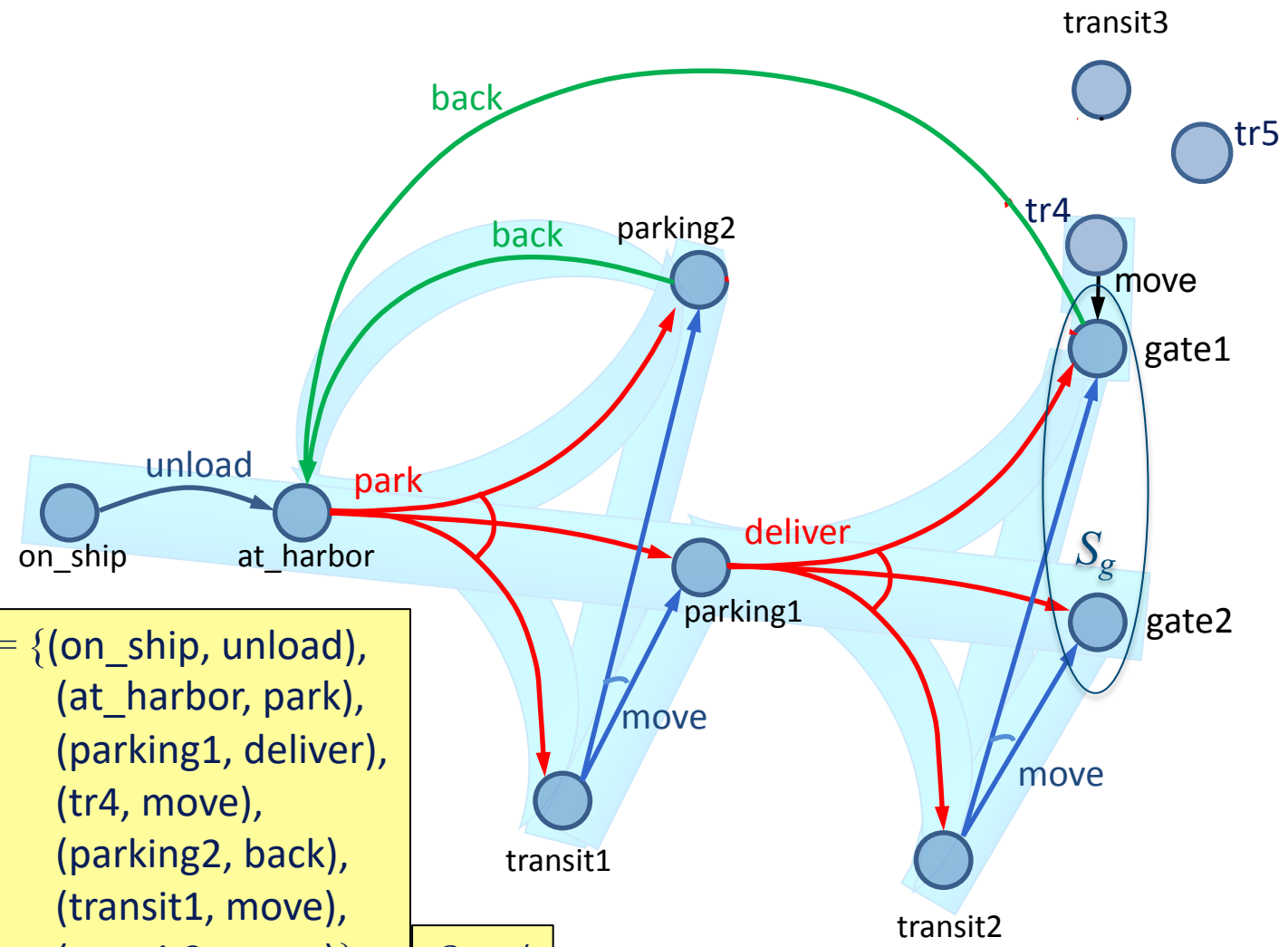


### Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

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      $\pi \leftarrow \pi \setminus \{(s', a)\}$   
     make  $a$  not applicable in  $s'$

## Example



$\pi = \{(on\_ship, unload),$   
 $(at\_harbor, park),$   
 $(parking1, deliver),$   
 $(tr4, move),$   
 $(parking2, back),$   
 $(transit1, move),$   
 $(transit2, move)\}$

$Q = \emptyset$

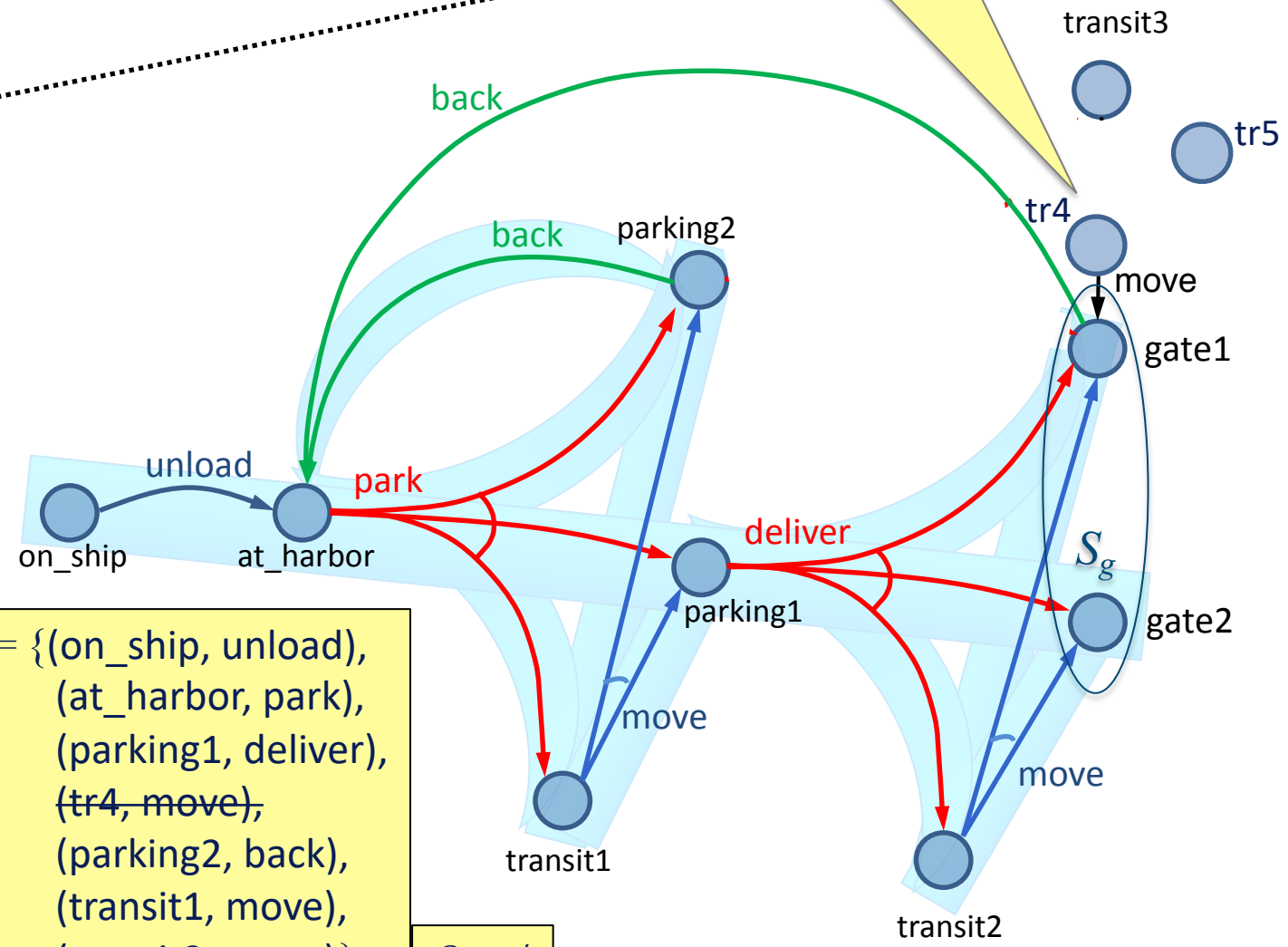
# Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

```

if  $s_0 \in S_g$  then return( $\emptyset$ )
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 $\pi \leftarrow \emptyset$ 
loop
   $Q \leftarrow leaves(s_0, \pi) \setminus S_g$ 
  if  $Q = \emptyset$  then do
     $\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$ 
    return( $\pi$ )
  select arbitrarily  $s \in Q$ 
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  else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do
     $\pi \leftarrow \pi \setminus \{(s', a)\}$ 
    make  $a$  not applicable in  $s'$ 
  
```

## Example

Remove (tr4, move) from  $\pi$  because  $\pi$  can't ever reach tr4



$\pi = \{(on\_ship, unload), (at\_harbor, park), (parking1, deliver), (\cancel{tr4}, \cancel{move}), (parking2, back), (transit1, move), (transit2, move)\}$

$Q = \emptyset$

## Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

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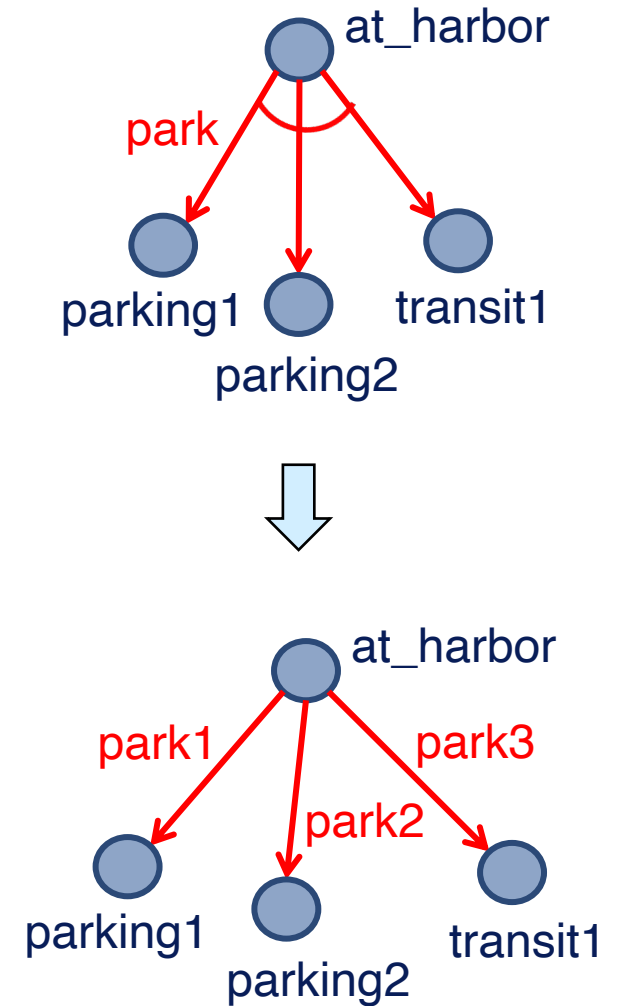
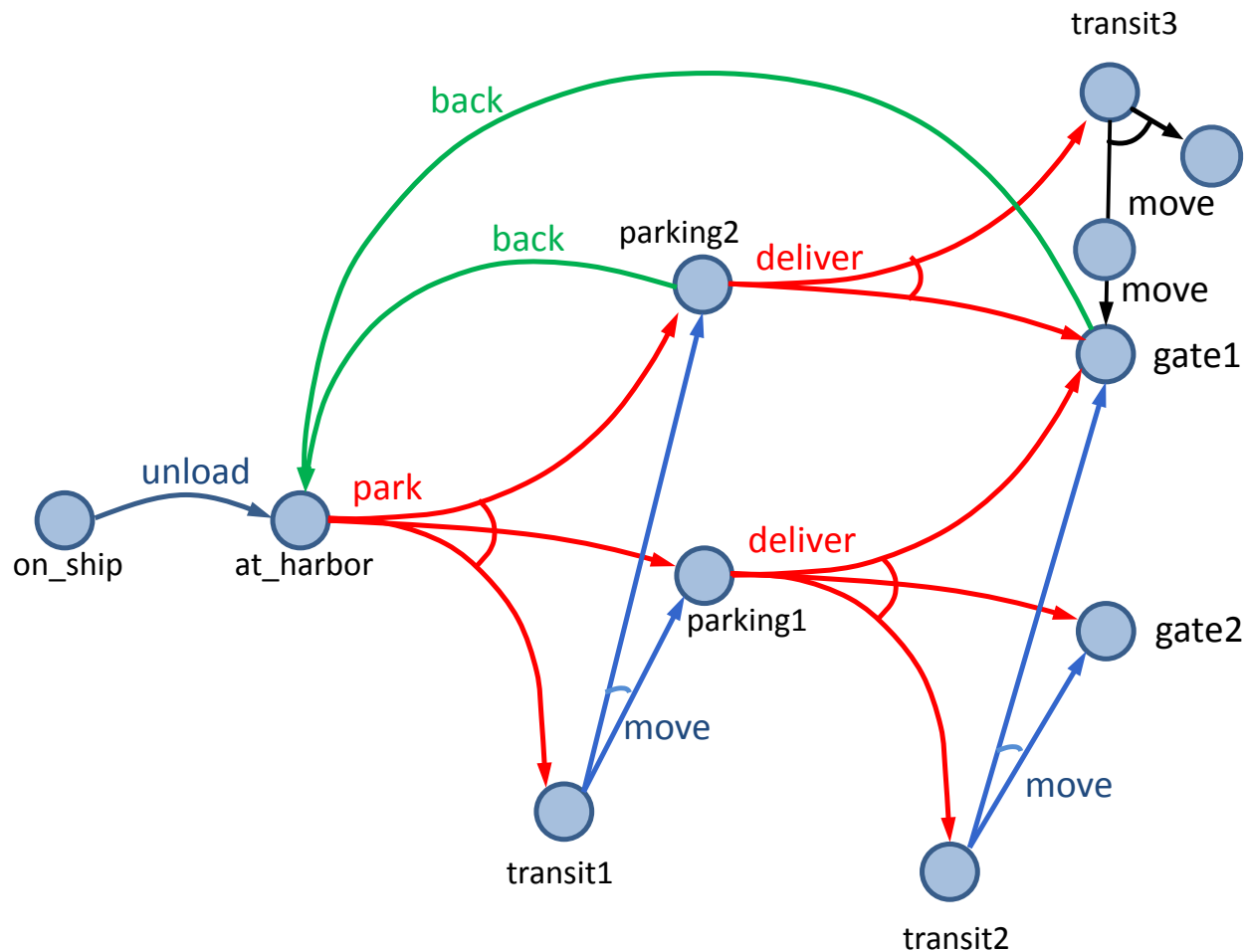
make  $a$  not applicable in  $s'$

## Discussion

- How to implement it?
  - ▶ Need implementation of Find-Solution
  - ▶ Need it to be very efficient
    - We'll call it many times
- Idea: instead of Find-Solution, use a classical planner
  - ▶ Any of the algorithms from Chapter 2
  - ▶ Efficient algorithms, search heuristics
- Need to convert the actions into something the classical planner can use ...

# Determinization

- Let  $a_i$  be a nondeterministic action with  $n$  possible outcomes
- *Determinization* of  $a_i = \{n \text{ deterministic actions, one for each outcome of } a_i\}$

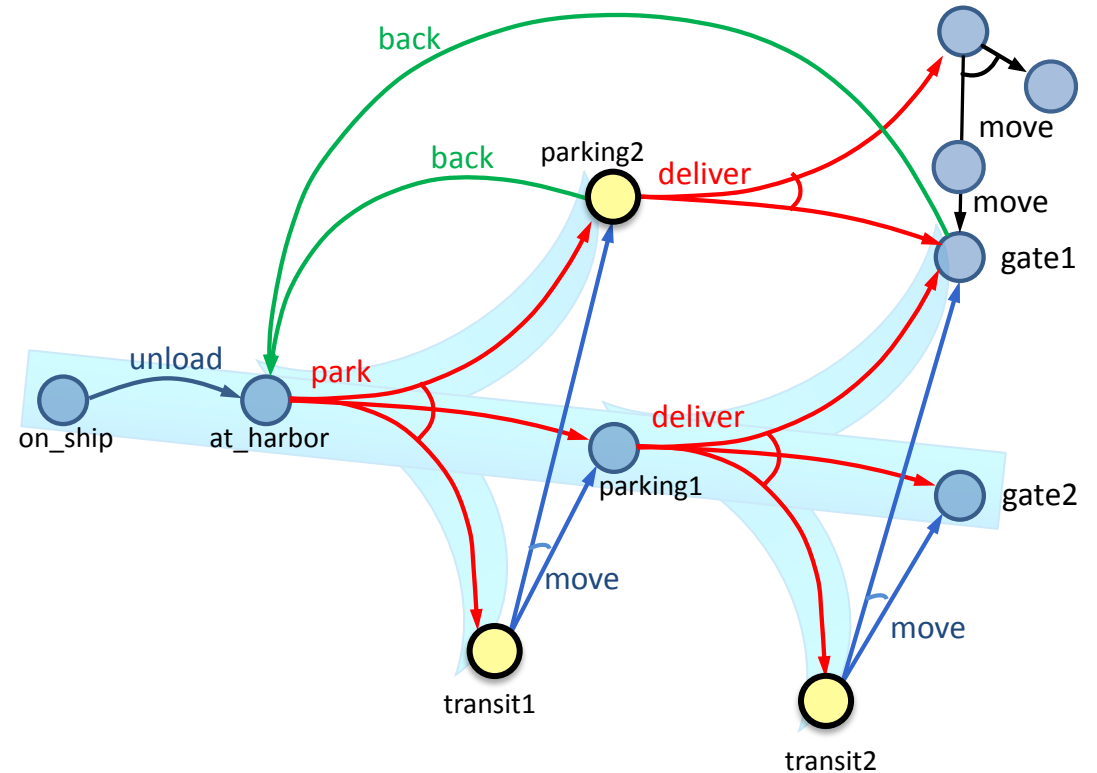
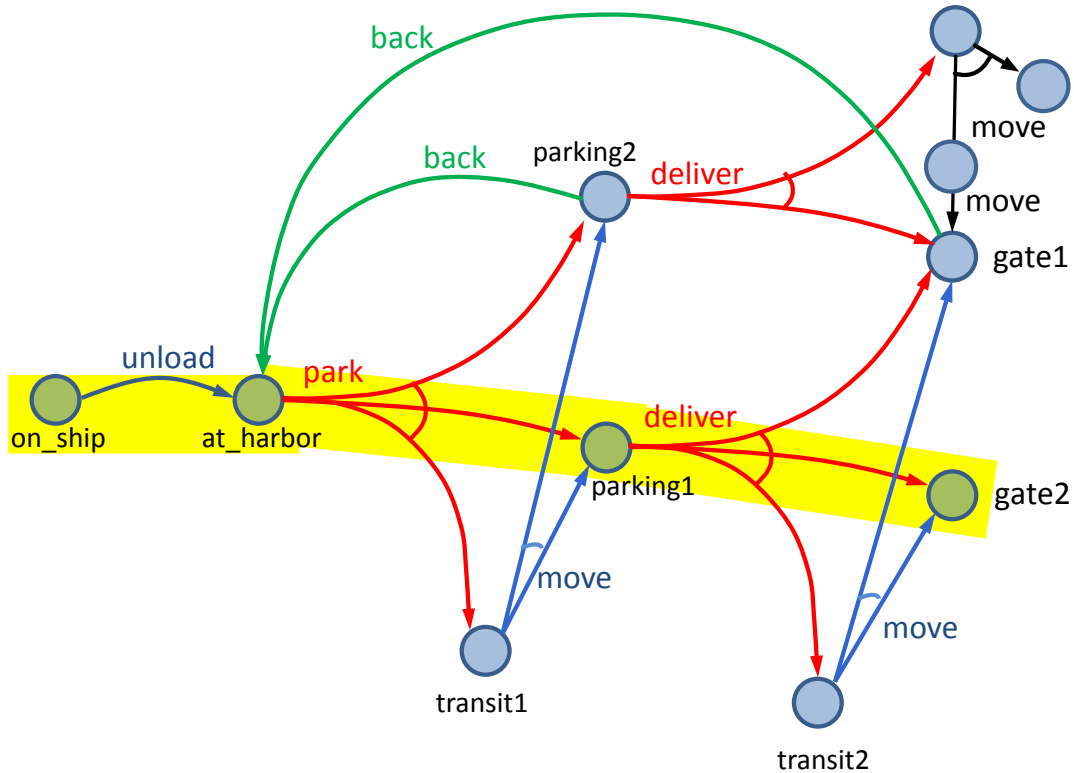


# Determinization

- Suppose a classical planner returns an acyclic plan  $p = \langle a_1, a_2, \dots, a_n \rangle$
- Actions and states:  $\langle s_0, a_1, s_1, a_2, s_2, a_3, \dots, a_n, s_n \rangle$
- Convert  $p$  to a policy  $\langle (s_0, a_1), (s_1, a_2), \dots, (s_{n-1}, a_n) \rangle$ 
  - ▶  $a_1$  = the nondeterministic action whose determinization includes  $a_i$

```

Plan2policy( $p = \langle a_1, \dots, a_n \rangle, s$ )
 $\pi \leftarrow \emptyset$ 
loop for  $i$  from 1 to  $n$  do
     $\pi \leftarrow \pi \cup (s, \text{det2nondet}(a_i))$ 
     $s \leftarrow \gamma_d(s, a_i)$ 
return  $\pi$ 
    
```





# Find-Safe-Solution-by-Determinization

Guided-Find-Safe-Solution ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

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$\pi \leftarrow \emptyset$

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if  $Q = \emptyset$  then do

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select arbitrarily  $s \in Q$

$\pi' \leftarrow \text{Find-Solution}(\Sigma, s, S_g)$

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select  $s \in Q$

$p' \leftarrow \text{Forward-search}(\Sigma_d, s, S_g)$

if  $p' \neq \text{fail}$  then do

$\pi' \leftarrow \text{Plan2policy}(p', s)$

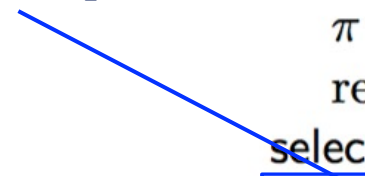
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$\pi \leftarrow \pi \setminus \{(s', a)\}$

make the actions in the determinization of  $a$  not applicable in  $s'$

Any classical  
planner that doesn't  
return cyclic plans



# Example

Find-Safe-Solution-by-Determinization ( $\Sigma, s_0, S_g$ )

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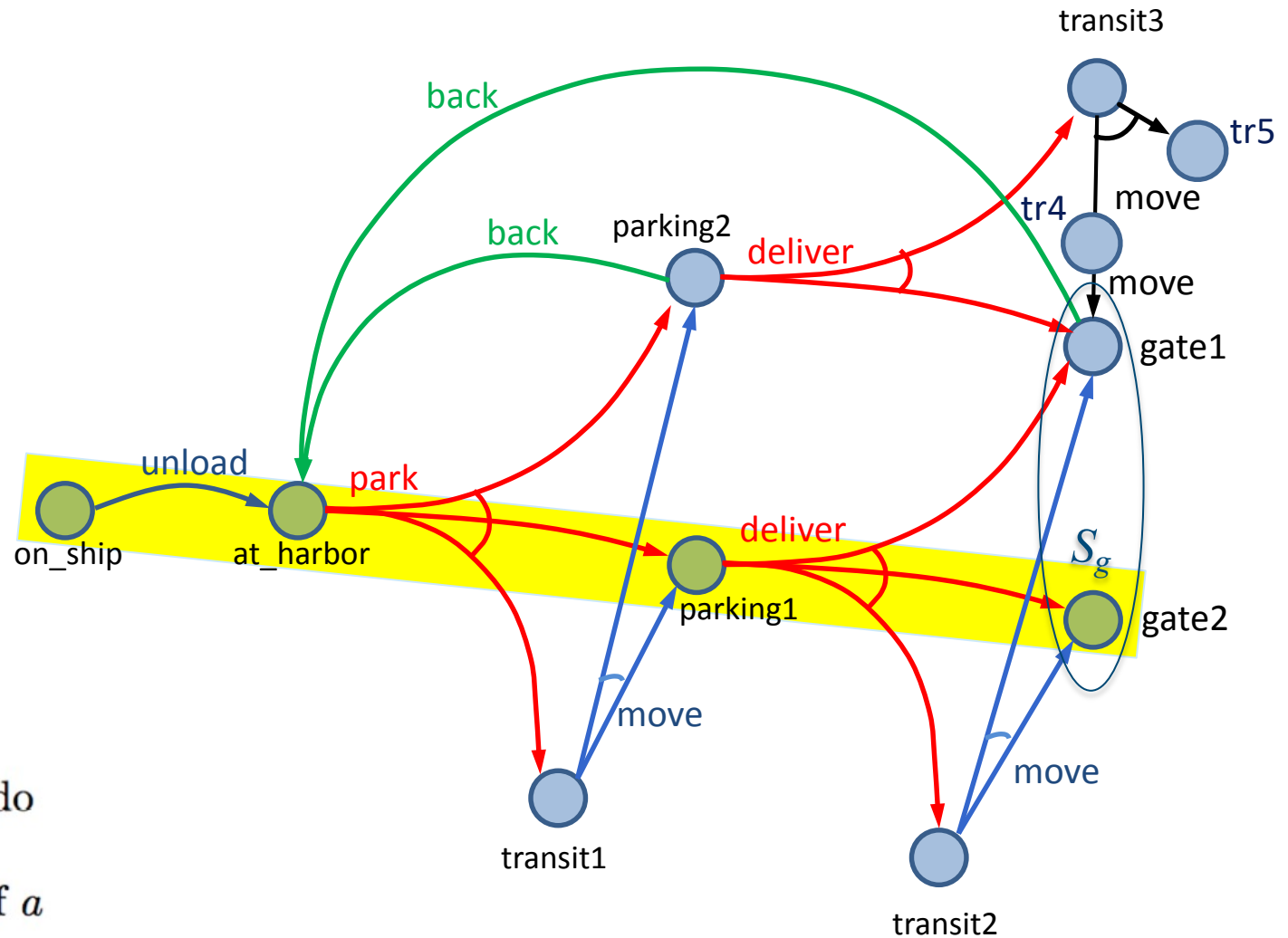
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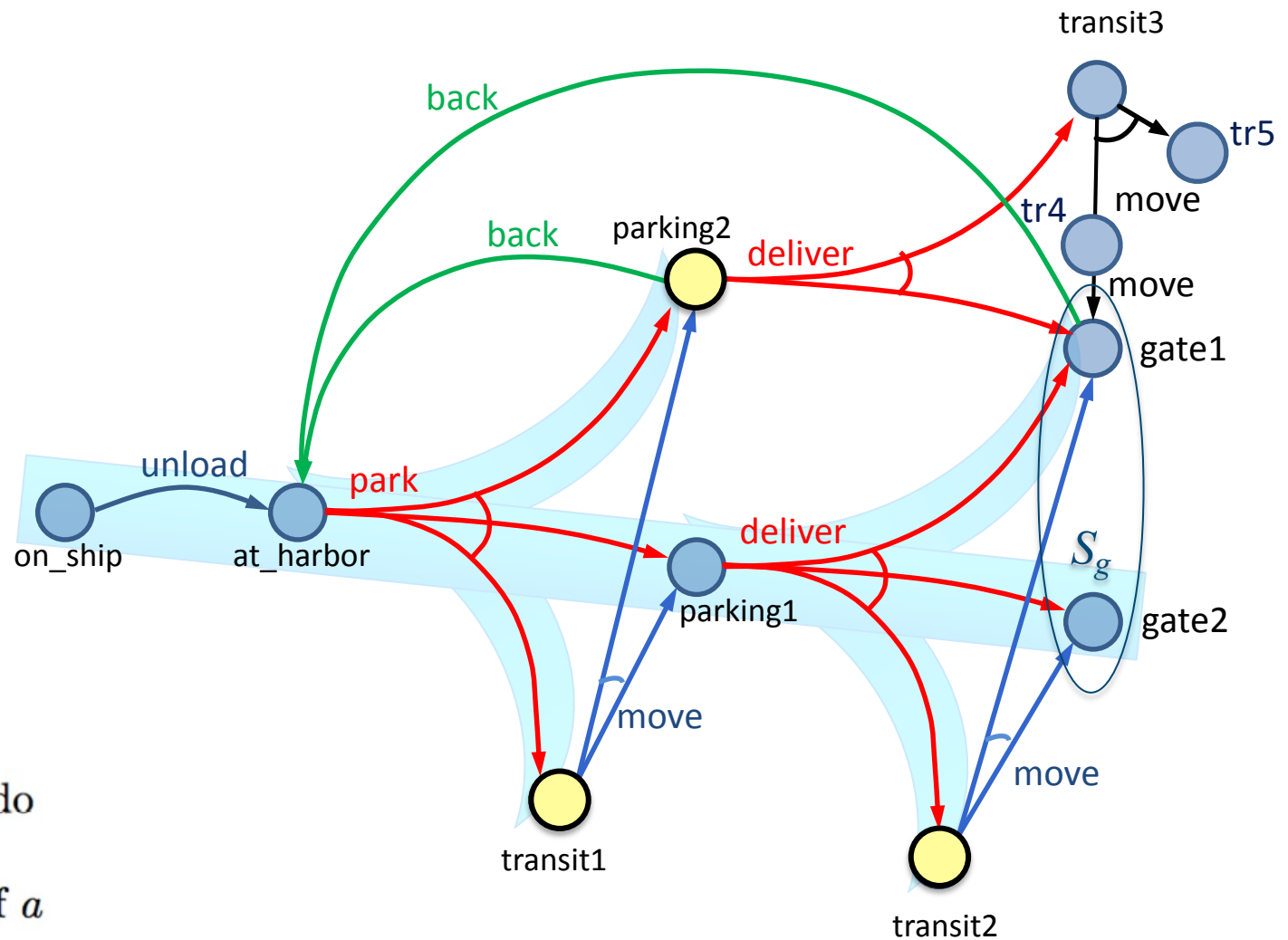
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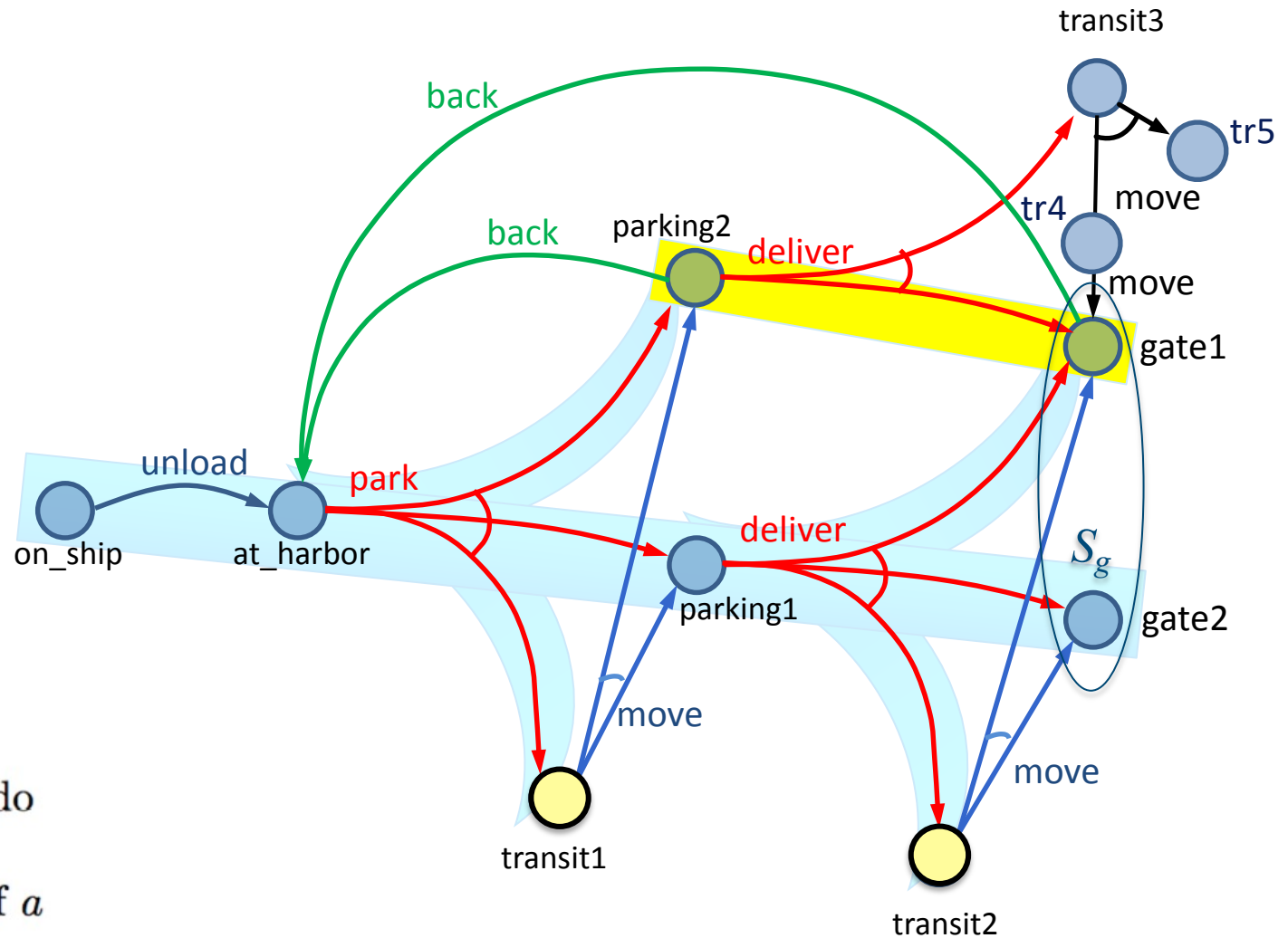
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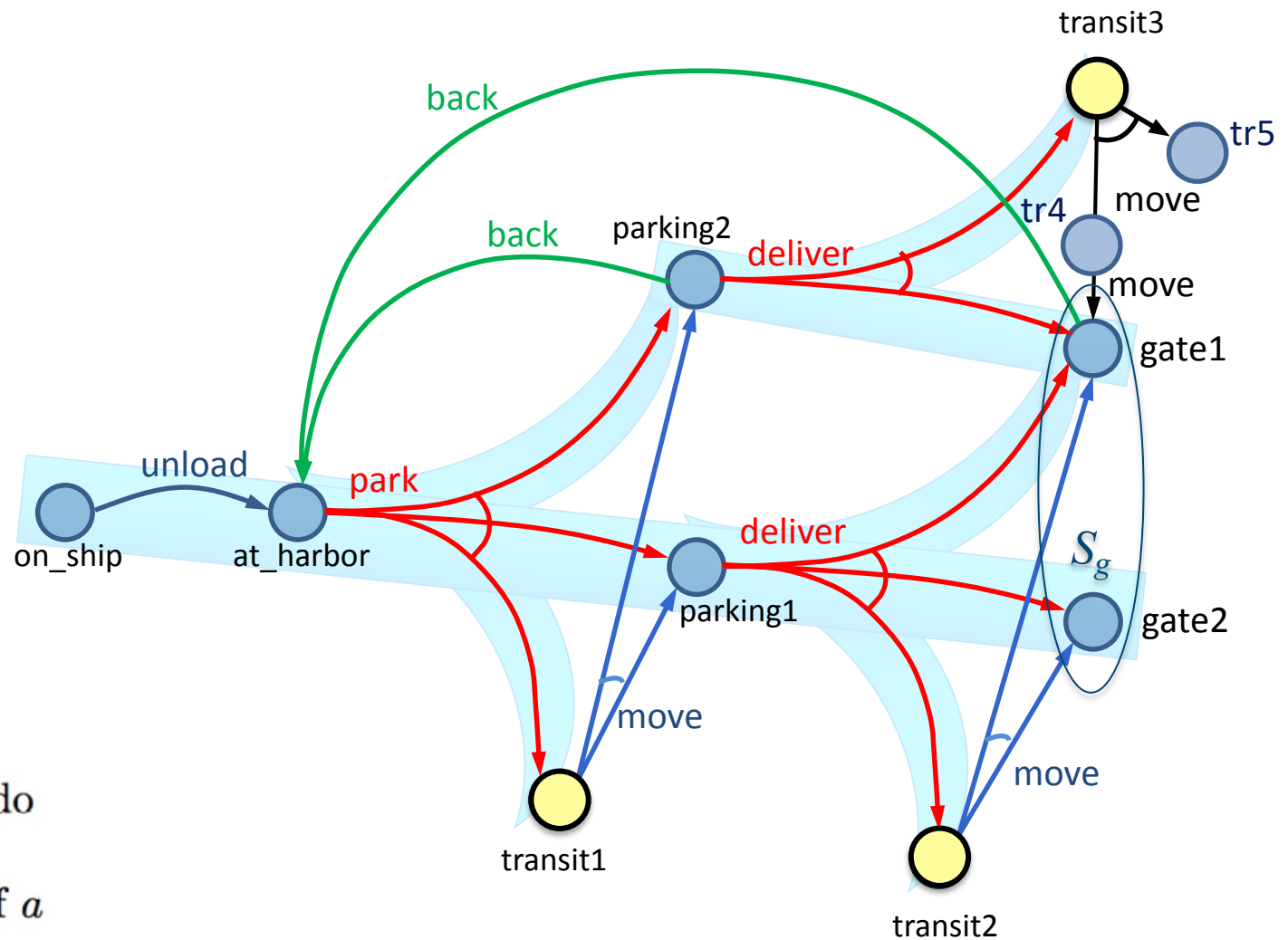
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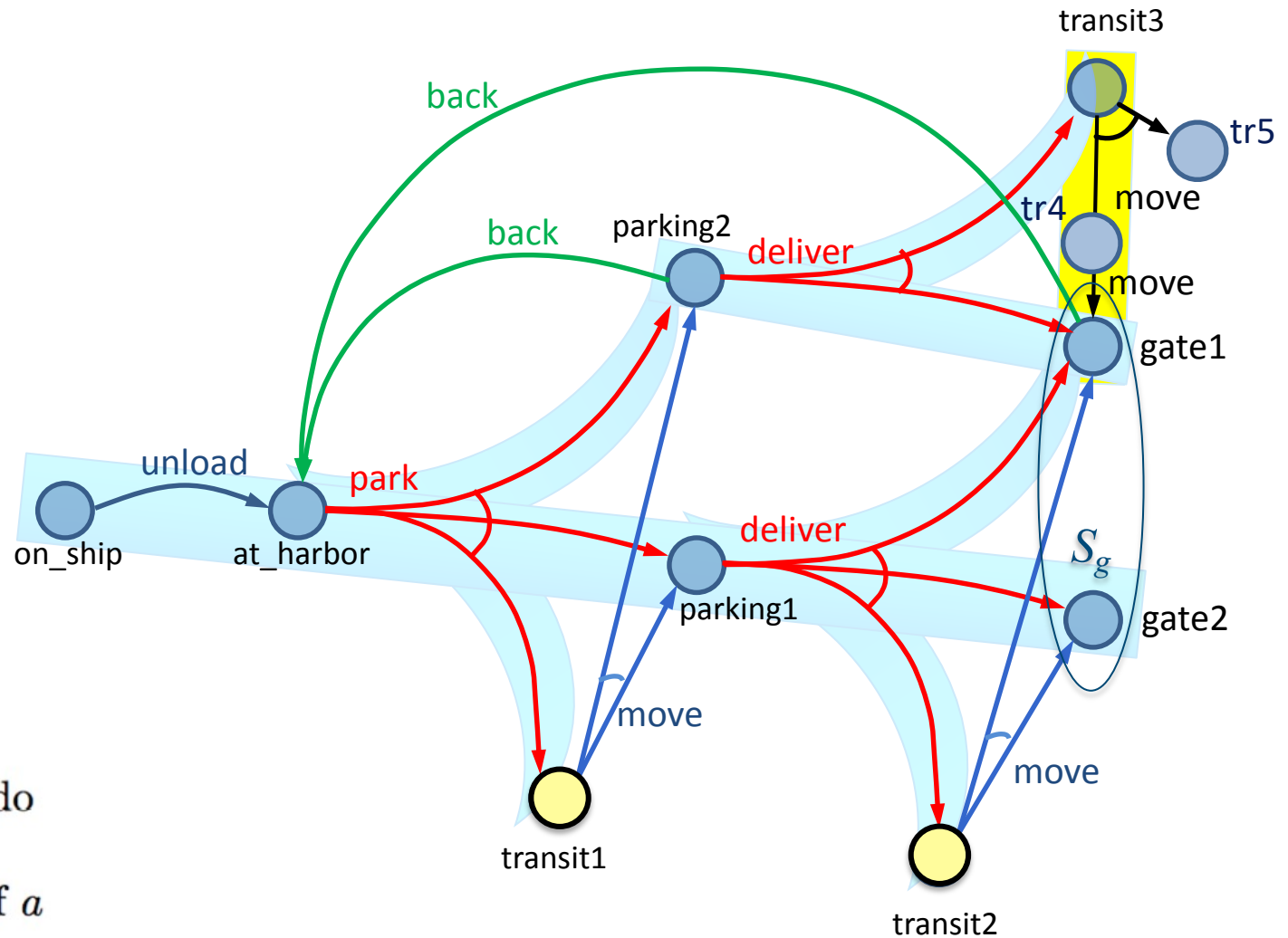
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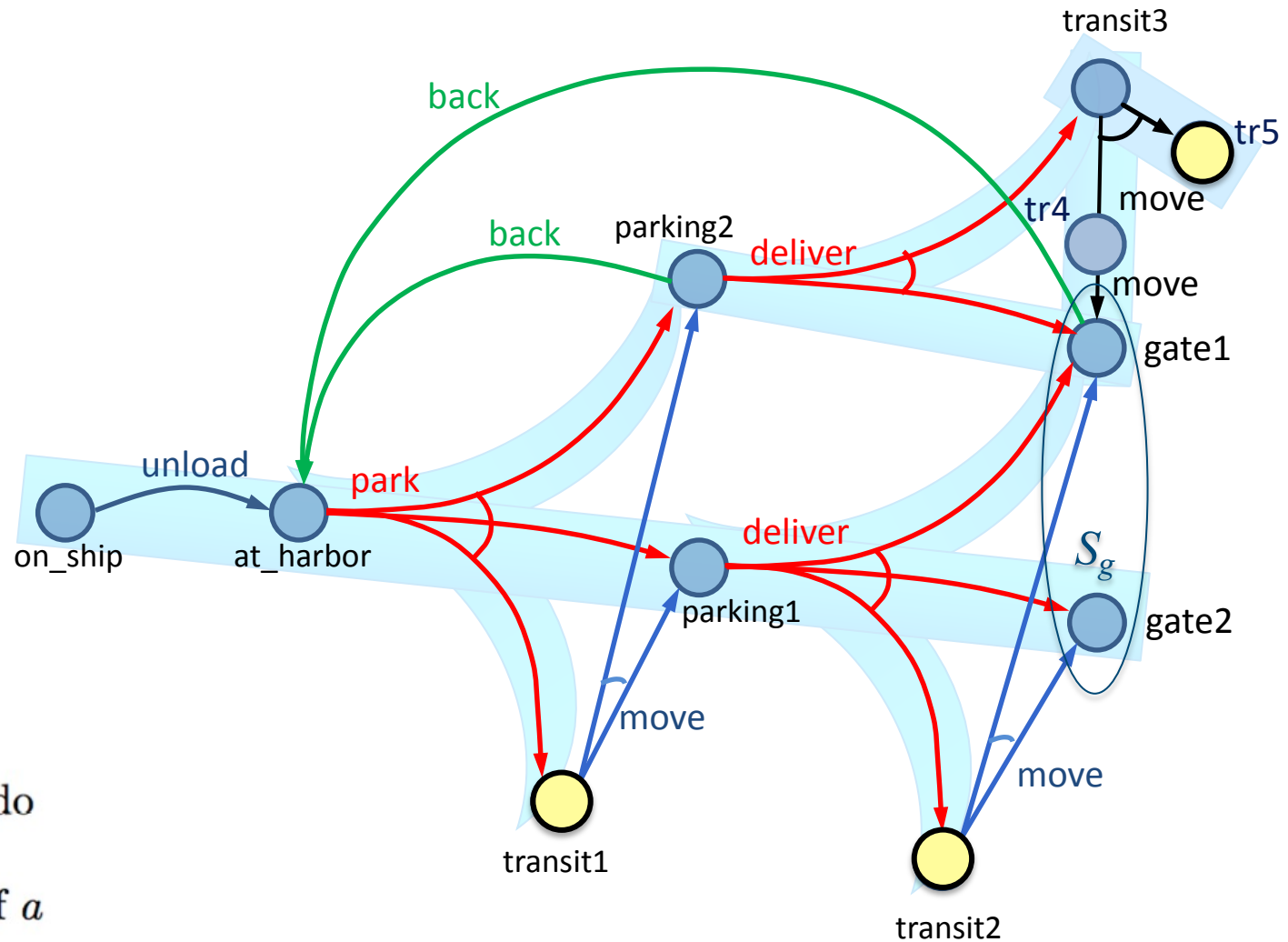
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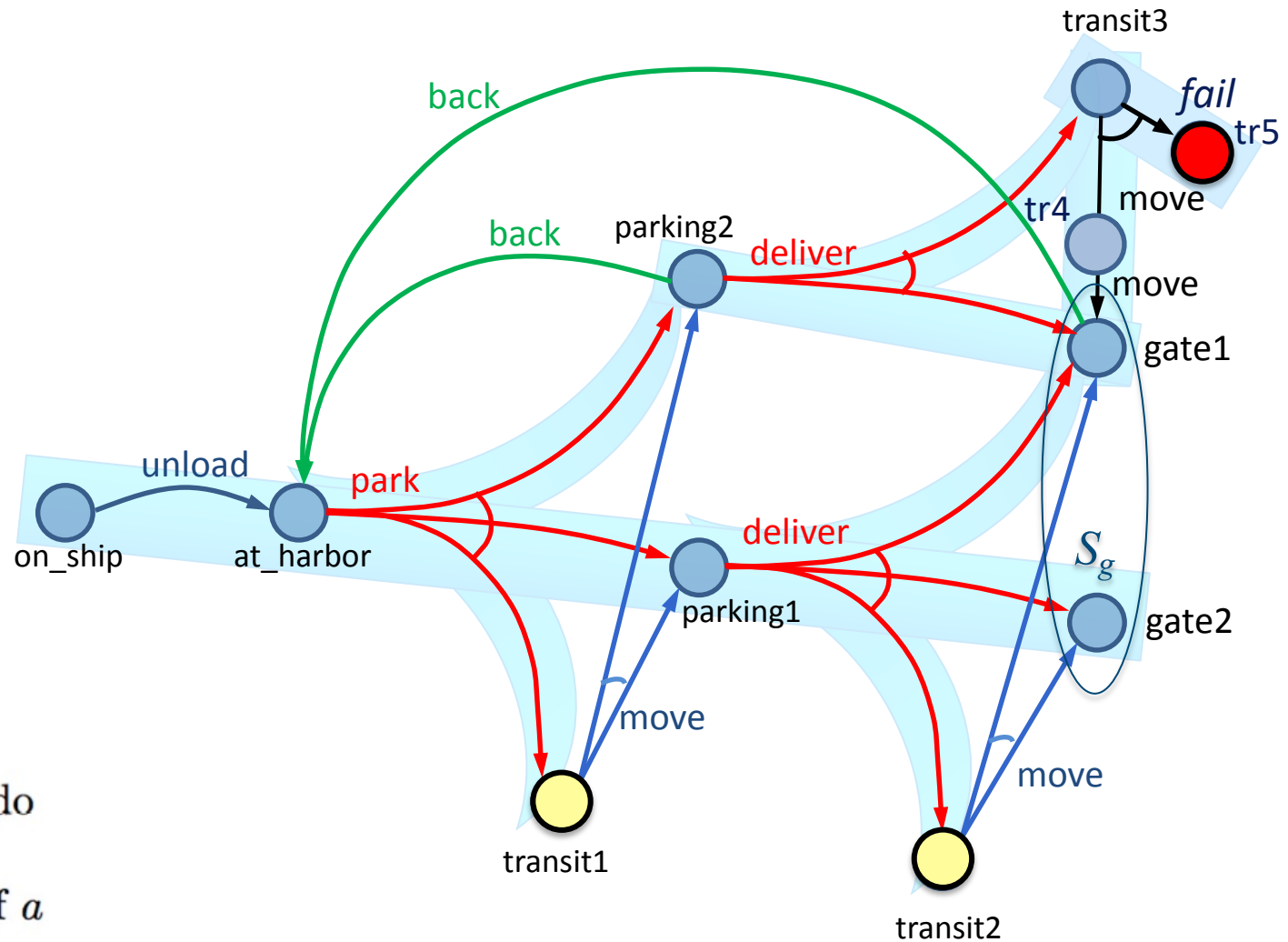
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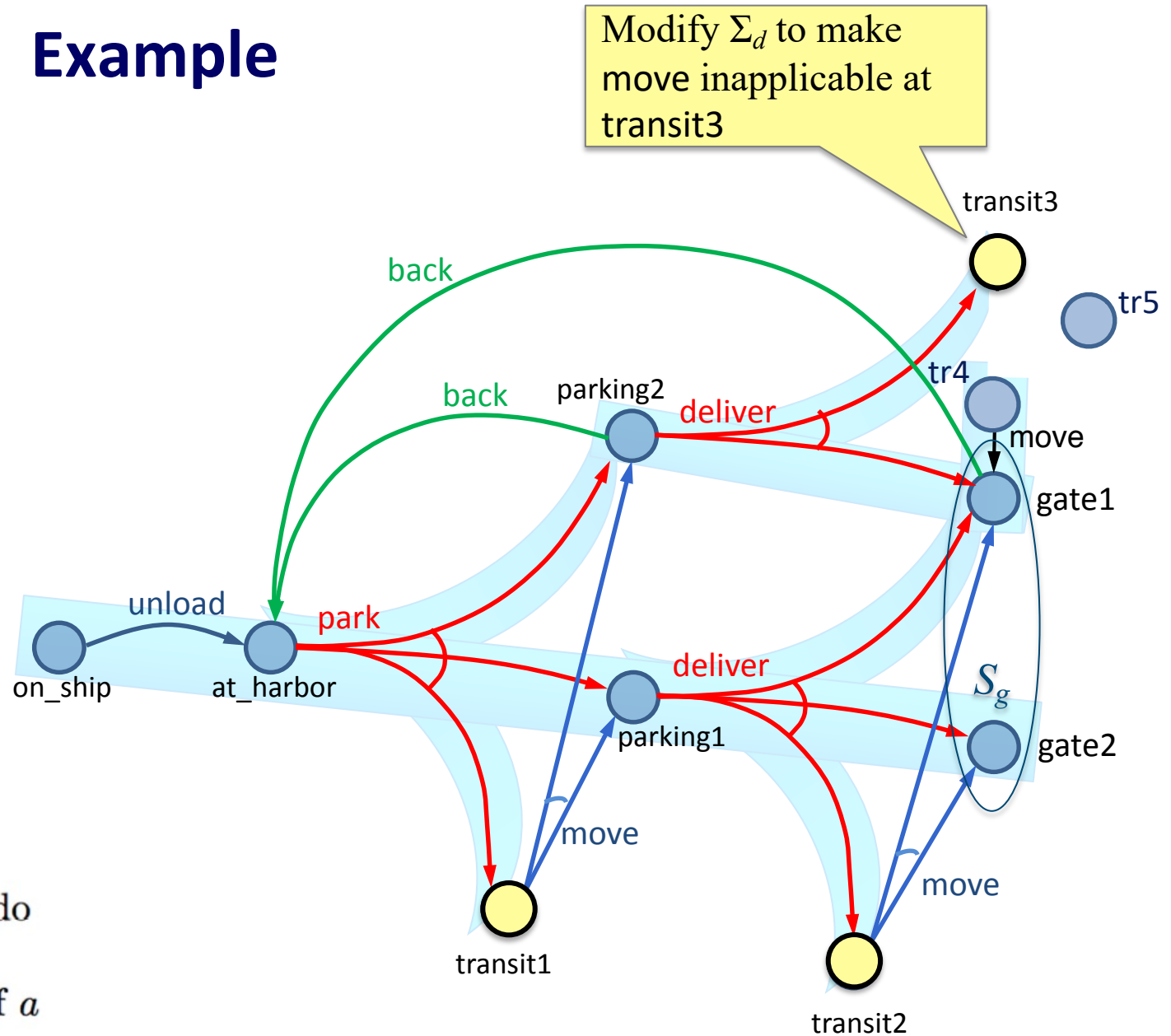
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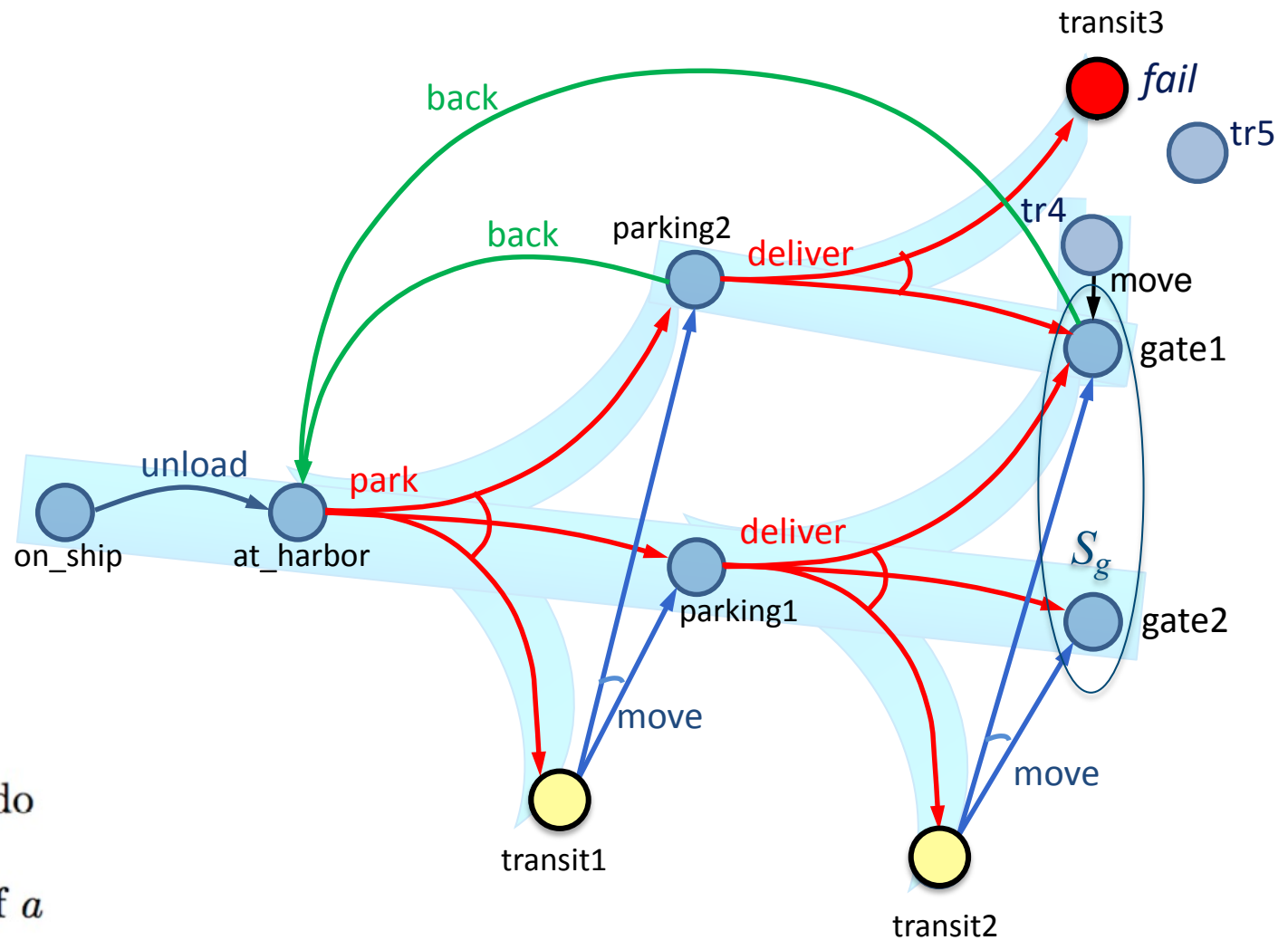
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# Example

Find-Safe-Solution-by-Determinization  $(\Sigma, s_0, S_g)$

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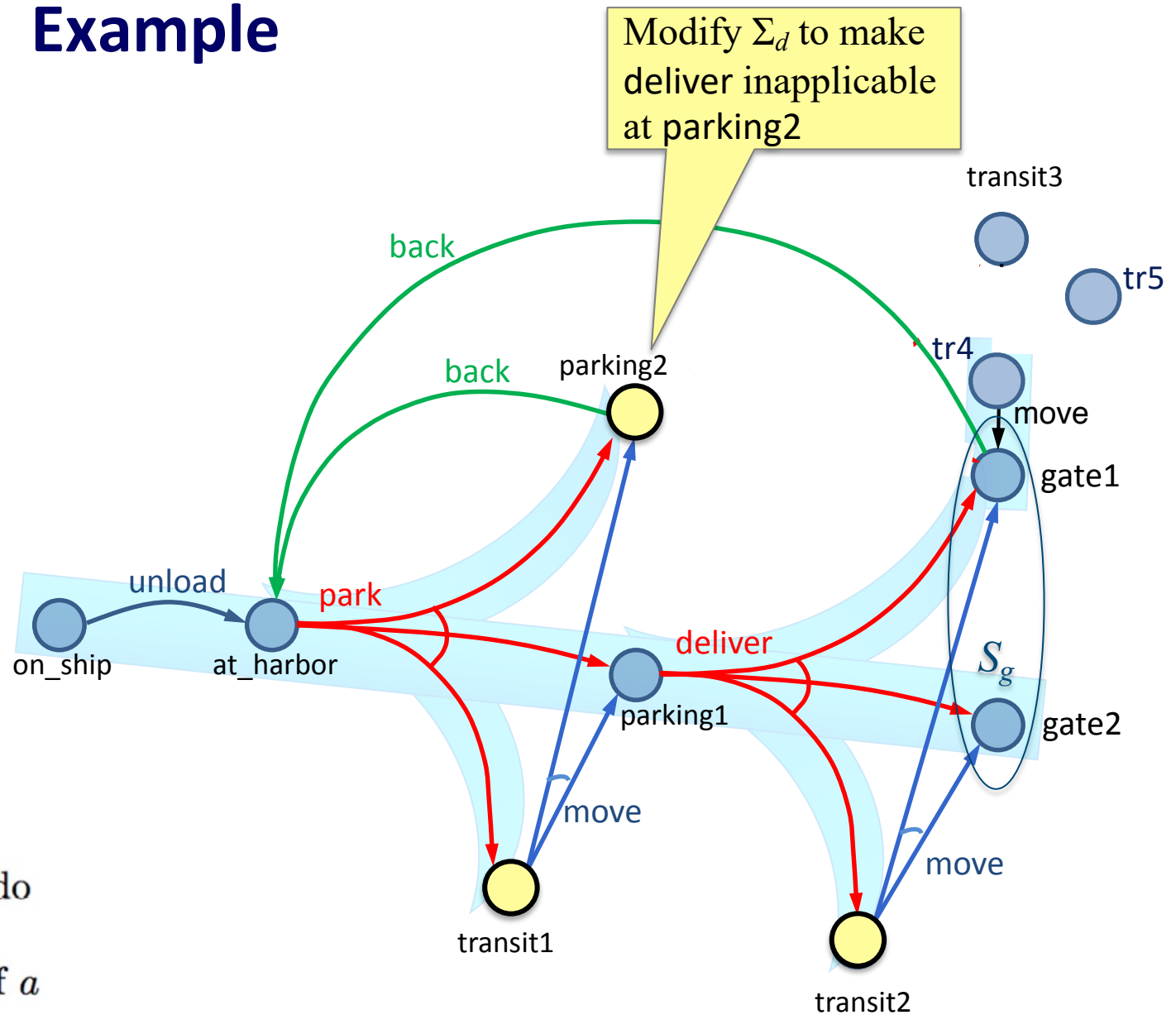
$\pi' \leftarrow \text{Plan2policy}(p', s)$

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else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make the actions in the determinization of  $a$   
not applicable in  $s'$



# Example

Find-Safe-Solution-by-Determinization ( $\Sigma, s_0, S_g$ )

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

$\Sigma_d \leftarrow \text{mk-deterministic}(\Sigma)$

loop

$Q \leftarrow \text{leaves}(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select  $s \in Q$

$p' \leftarrow \text{Forward-search}(\Sigma_d, s, S_g)$

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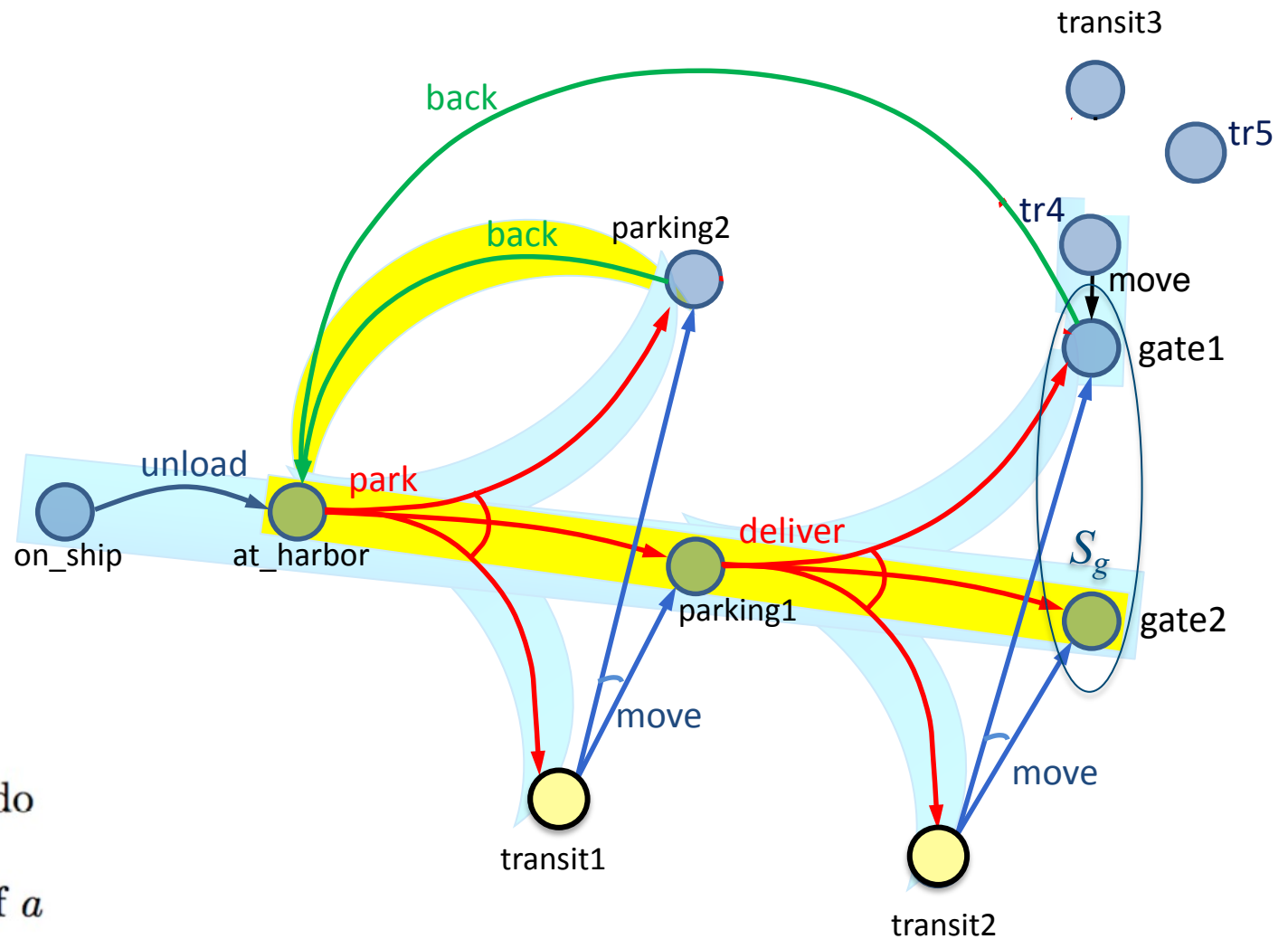
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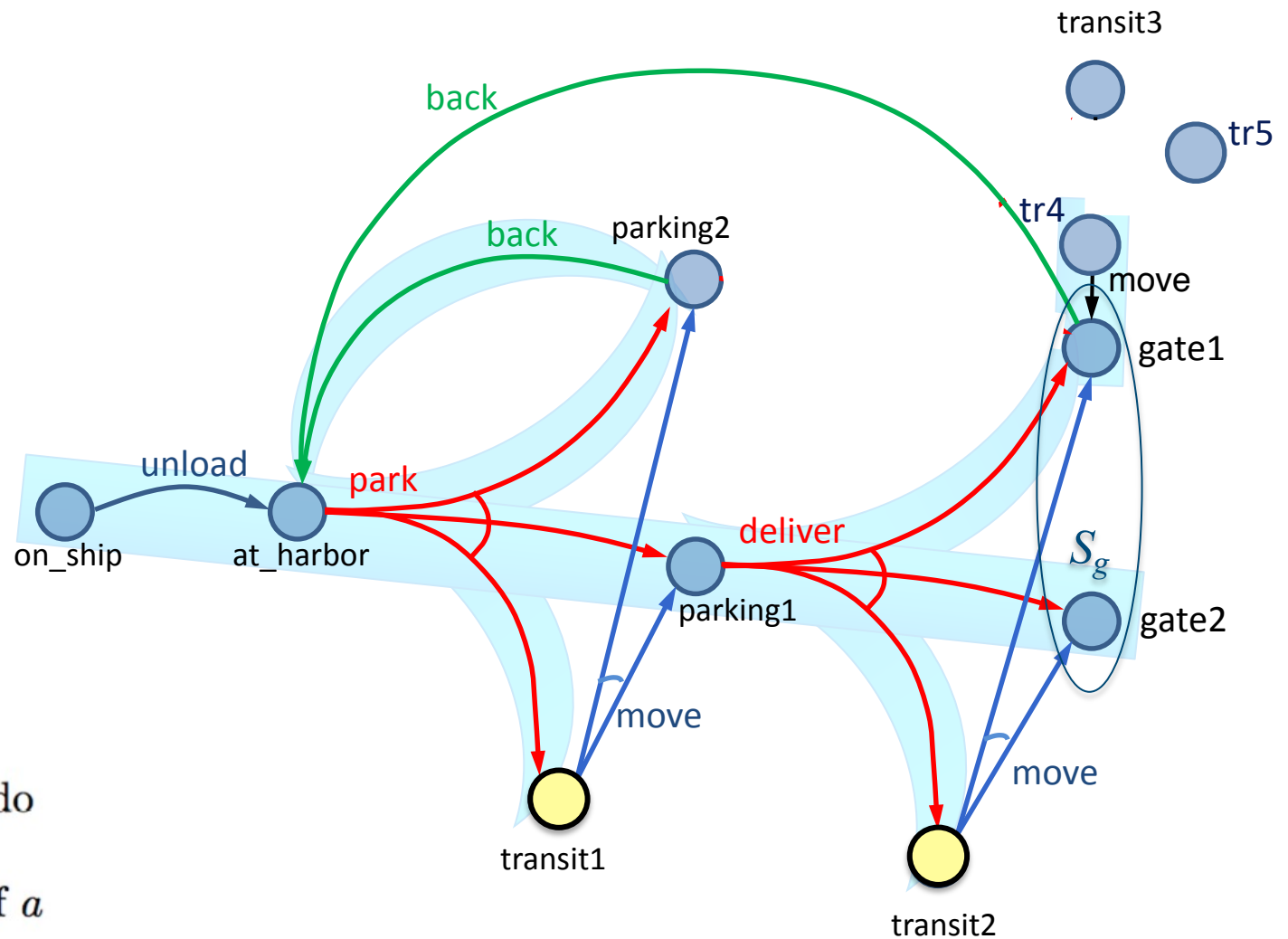
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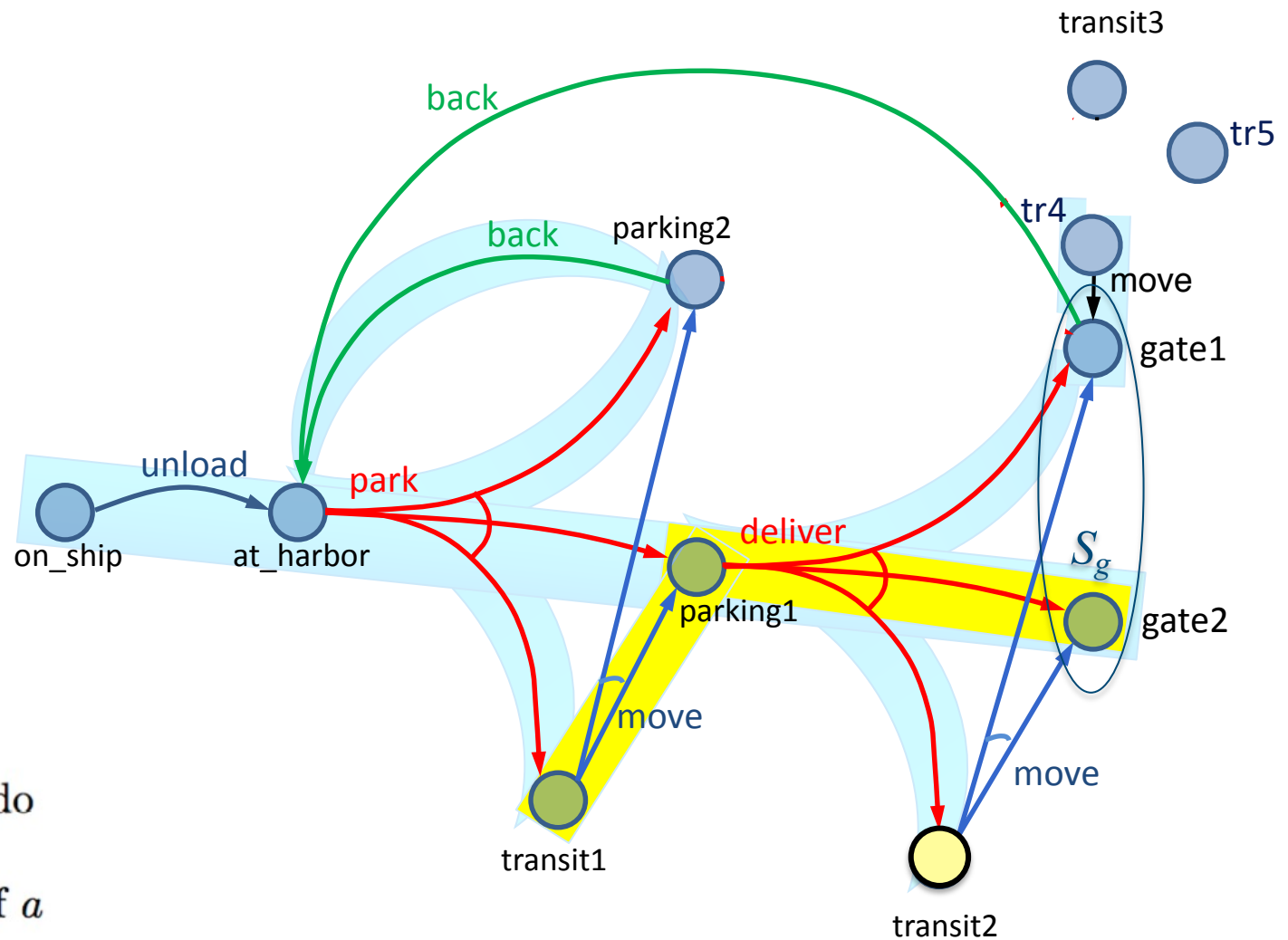
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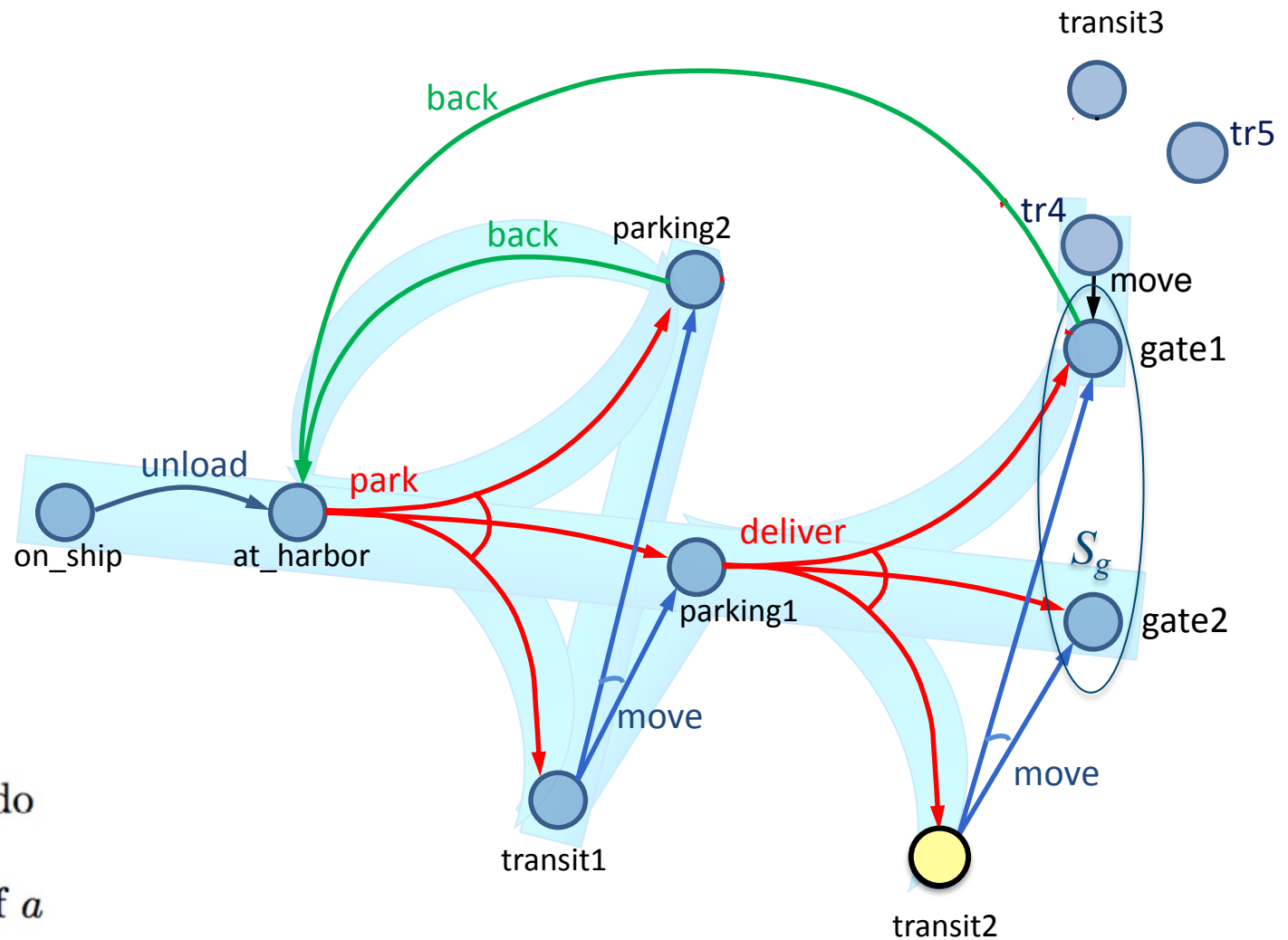
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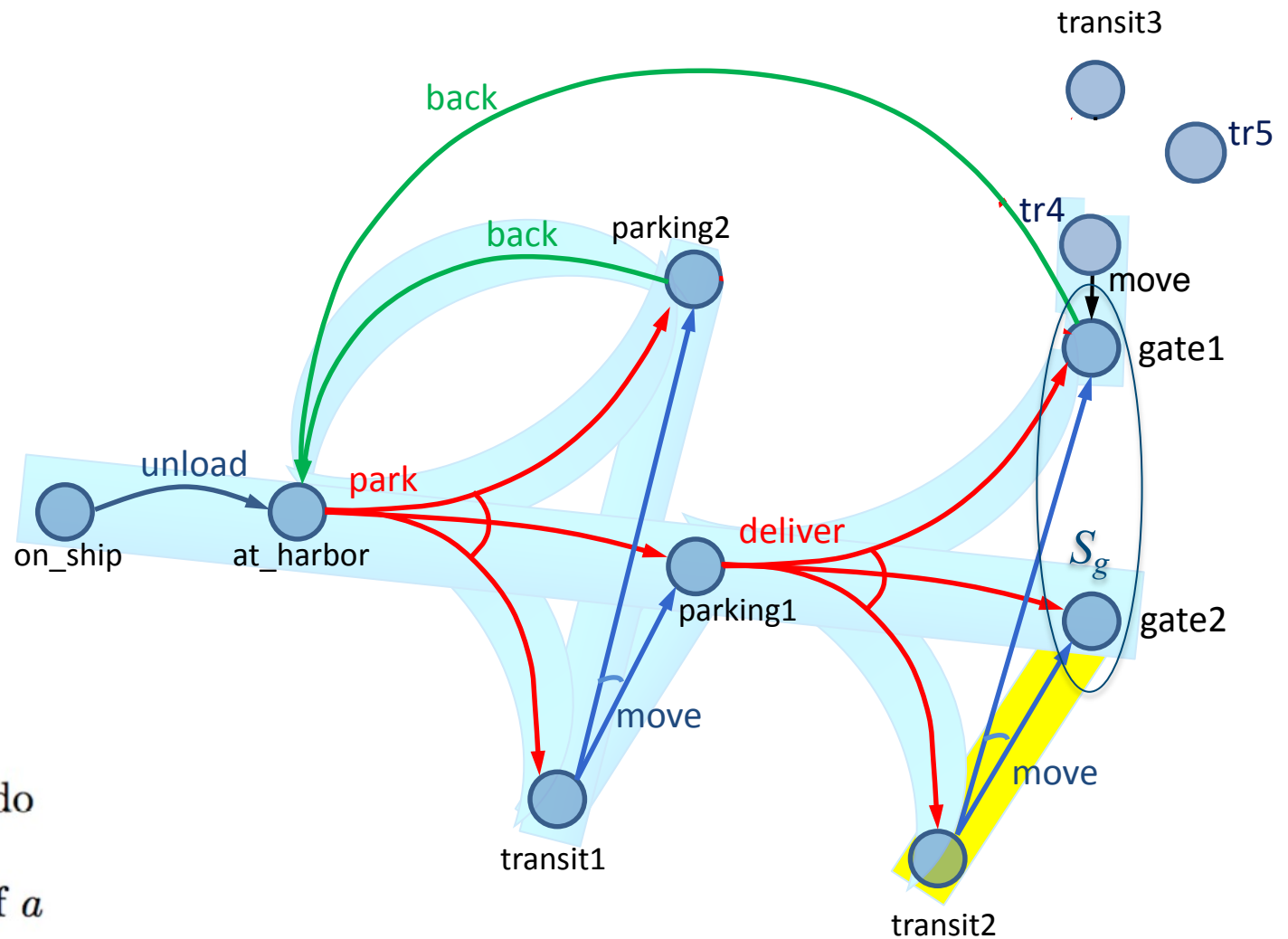
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Find-Safe-Solution-by-Determinization ( $\Sigma, s_0, S_g$ )

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if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select  $s \in Q$

$p' \leftarrow \text{Forward-search}(\Sigma_d, s, S_g)$

if  $p' \neq \text{fail}$  then do

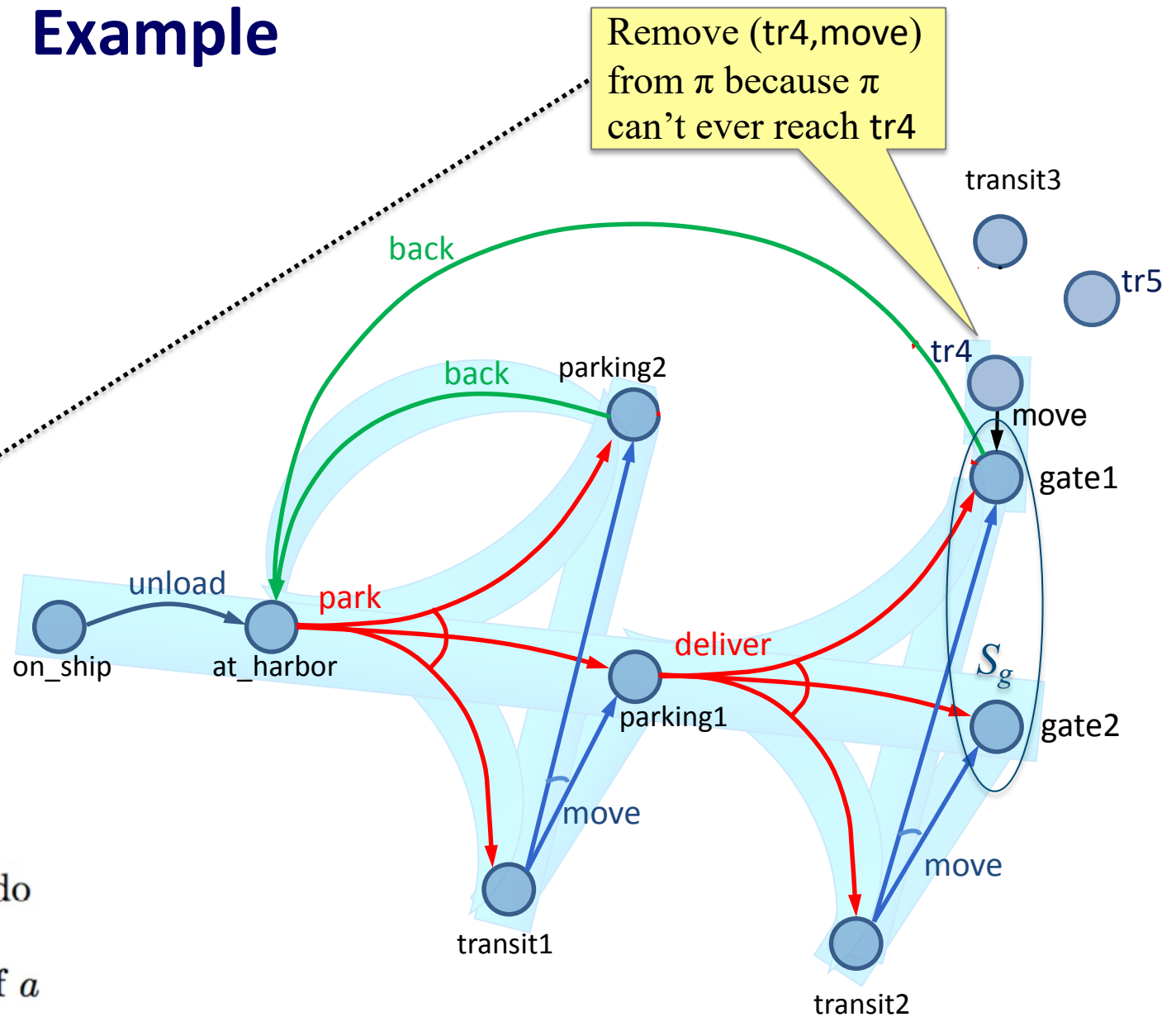
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else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make the actions in the determinization of  $a$   
not applicable in  $s'$



# Making Actions Inapplicable

Find-Safe-Solution-by-Determinization  $(\Sigma, s_0, S_g)$

if  $s_0 \in S_g$  then return( $\emptyset$ )

if  $Applicable(s_0) = \emptyset$  then return(failure)

$\pi \leftarrow \emptyset$

$\Sigma_d \leftarrow \text{mk-deterministic}(\Sigma)$

loop

$Q \leftarrow \text{leaves}(s_0, \pi) \setminus S_g$

if  $Q = \emptyset$  then do

$\pi \leftarrow \pi \setminus \{(s, a) \in \pi \mid s \notin \hat{\gamma}(s_0, \pi)\}$

return( $\pi$ )

select  $s \in Q$

$p' \leftarrow \text{Forward-search}(\Sigma_d, s, S_g)$

if  $p' \neq \text{fail}$  then do

$\pi' \leftarrow \text{Plan2policy}(p', s)$

$\pi \leftarrow \pi \cup \{(s, a) \in \pi' \mid s \notin \text{Dom}(\pi)\}$

else for every  $s'$  and  $a$  such that  $s \in \gamma(s', a)$  do

$\pi \leftarrow \pi \setminus \{(s', a)\}$

make the actions in the determinization of  $a$   
not applicable in  $s'$

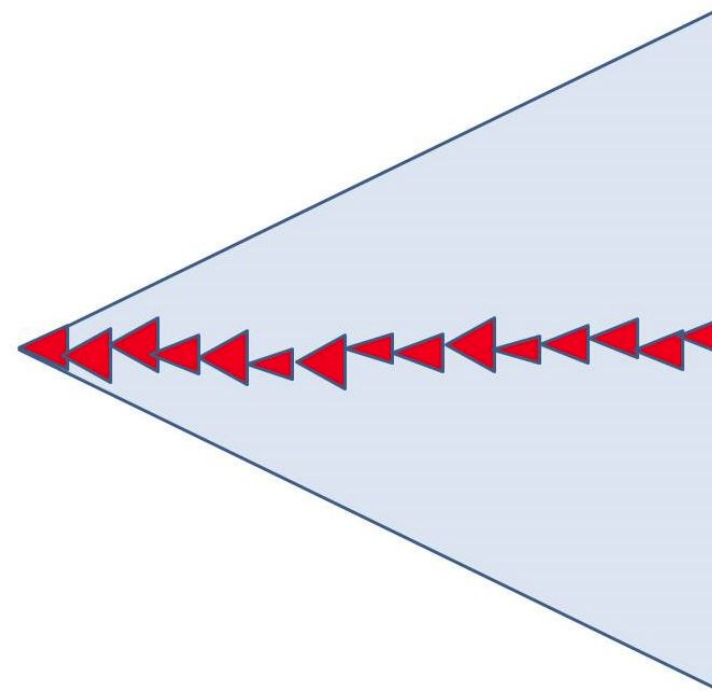
- Modify  $\Sigma_d$  to make  $a$  inapplicable at  $s$ 
  - worst-case exponential time
- Better: hash table of bad state-action pairs
  - For every  $(s', a)$  such that  $s \in \gamma(s', a)$ ,  
 $Bad[s'] \leftarrow Bad[s'] \cup \text{determinization}(a)$
  - Modify classical planner to take the table as an argument
    - if  $s$  is current state, only choose actions in  $Applicable(s) \setminus Bad[s]$

# Skip Ahead

- Several topics I'll skip for now
  - will come back later if there's time
  - ▶ Other kinds of search algorithms
    - min-max search
  - ▶ Symbolic model checking techniques
    - Backward search
    - BDD representation
      - ▶ Reduce search-space size by planning over sets of states

## 5.6 Online Approaches

- Motivation
  - ▶ Planning models are approximate – execution seldom works out as planned
  - ▶ Large problems may require too much planning time
- 2<sup>nd</sup> motivation even more stronger in nondeterministic domains
  - ▶ Nondeterminism makes planning exponentially harder
    - Exponentially more time, exponentially larger policies



Offline vs Runtime  
Search Spaces



# Online Approaches

- Need to identify *good* actions without exploring entire search space
  - ▶ Can be done using heuristic estimates
- Some domains are *safely explorable*
  - ▶ Safe to create partial plans, because goal states are always reachable
- In domains with dead-ends, partial planning won't guarantee success
  - ▶ Can get trapped in dead ends that we would have detected if we had planned fully
    - No applicable actions
      - ▶ robot's battery goes dead
    - Applicable actions, but caught in a loop
      - ▶ robot goes into a collection of rooms from which there's no exit
  - ▶ But partial planning can still make success more likely

# Lookahead-Partial-Plan

- Adaptation of Run-Lazy-Lookahead (Chapter 2)
- Lookahead is any planning algorithm that returns a policy  $\pi$ 
  - ▶  $\pi$  may be partial or unsafe solution
- Execute  $\pi$  as far as it will go, then call Lookahead again

Lookahead-Partial-Plan( $\Sigma, s_0, S_g$ )

$s \leftarrow s_0$

while  $s \notin S_g$  and  $\text{Applicable}(s) \neq \emptyset$  do

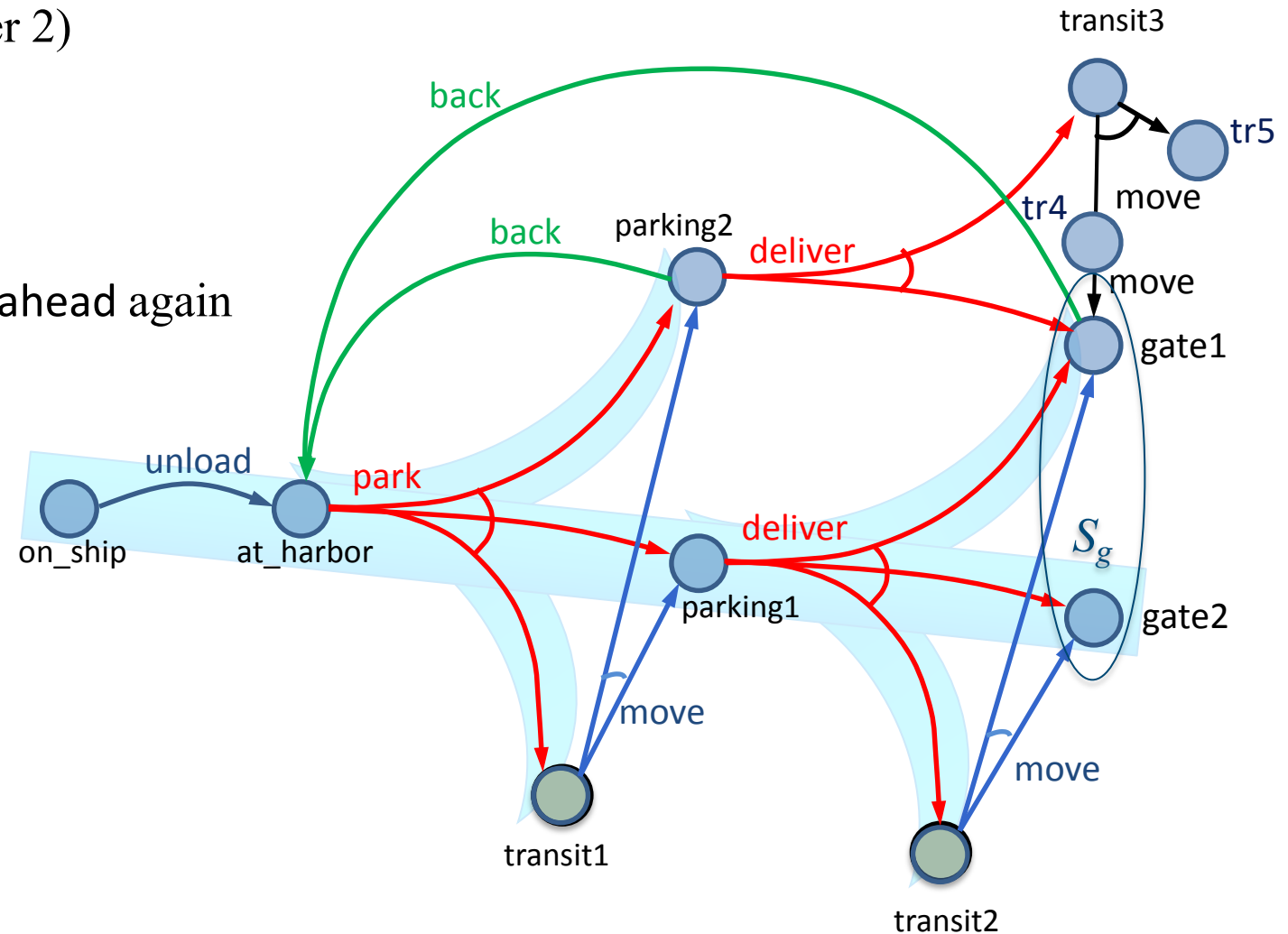
$\pi \leftarrow \text{Lookahead}(s, \theta)$

if  $\pi = \emptyset$  then return failure

else do

perform partial plan  $\pi$

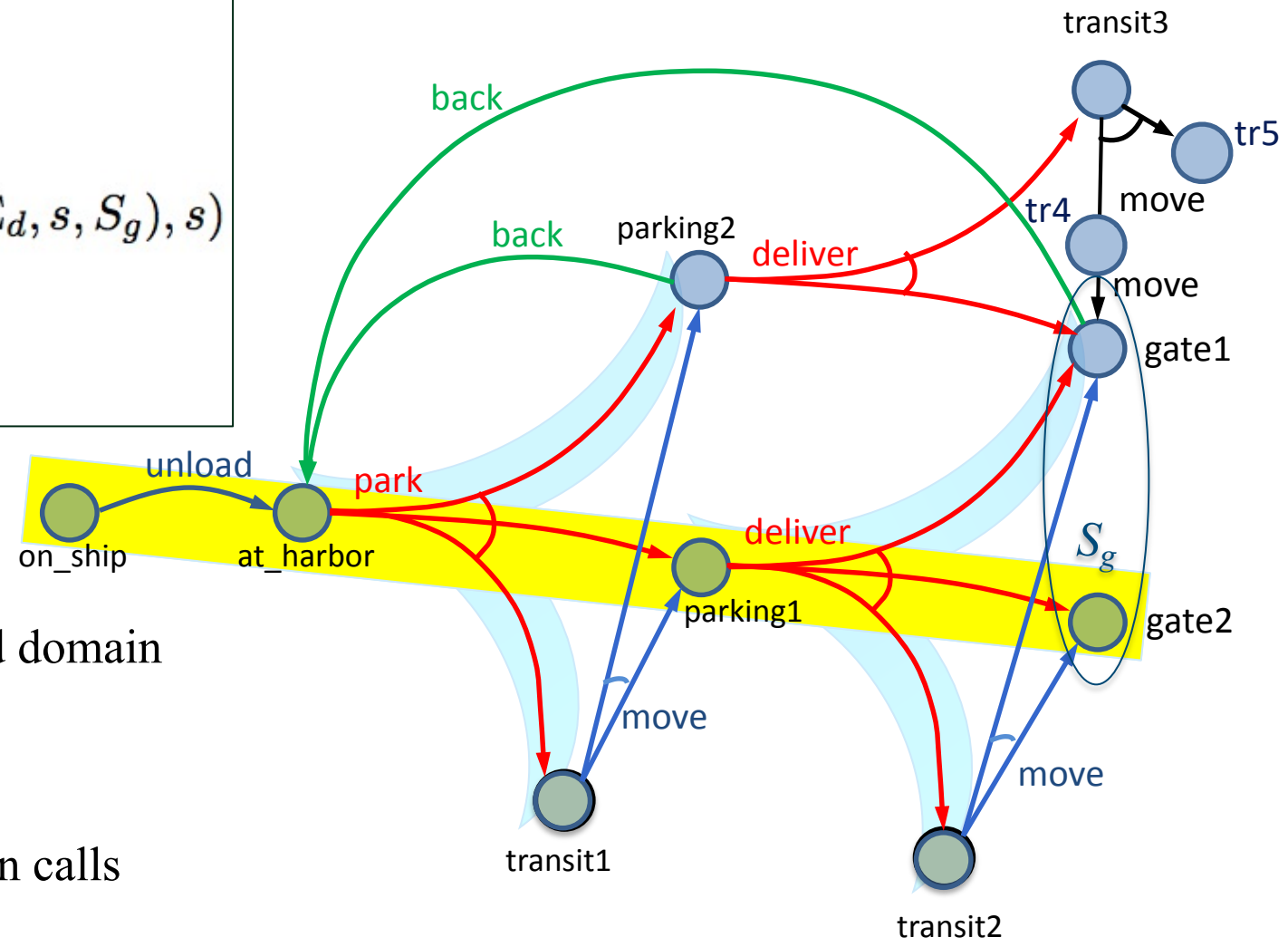
$s \leftarrow$  observe current state



# FS-Replan

```

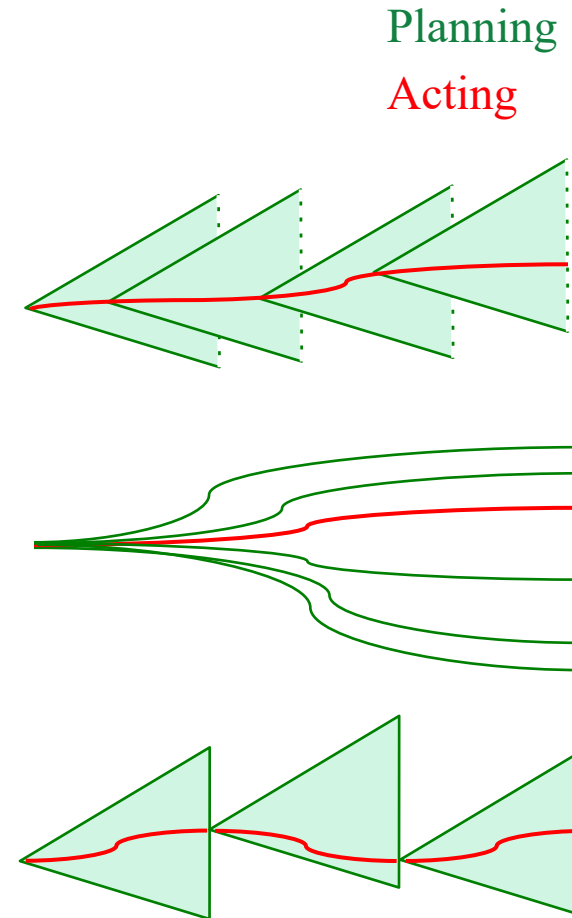
FS-Replan ( $\Sigma, s, S_g$ )
 $\pi_d \leftarrow \emptyset$ 
while  $s \notin S_g$  and  $\text{Applicable}(s) \neq \emptyset$  do
  if  $\pi_d$  undefined for  $s$  then do
     $\pi_d \leftarrow \text{Plan2policy}(\text{Forward-search}(\Sigma_d, s, S_g), s)$ 
    if  $\pi_d = \text{failure}$  then return failure
  perform action  $\pi_d(s)$ 
   $s \leftarrow$  observe resulting state
    
```



- Adaptation of Lookahead-Partial-Plan
  - ▶ Calls classical planner on determinized domain
  - ▶ Gets plan, converts converts to policy
    - Unsafe solution
  - ▶ Executes policy as far as it will go, then calls classical planner again

# More Possibilities for Lookahead

- What if Lookahead doesn't have time to run to completion?
  - ▶ Can use the same techniques we discussed in Chapter 3
    - Receding horizon
    - Sampling
    - Subgoaling
    - Iterative deepening
  - ▶ A few others ...



# More Possibilities for Lookahead

- *Full horizon, limited breadth:*
  - ▶ look for solution that works for *some* of the outcomes
  - ▶ E.g., modify Find-Acyclic-Solution to examine  $i$  outcomes of each action

- *Iterative broadening:*
  - for  $i = 1$  by 1 until time runs out
  - look for a solution that handles  $i$  outcomes per action

Find-Acyclic-Solution  $(\Sigma, s_0, S_g)$

$\pi \leftarrow \emptyset$

$Frontier \leftarrow \{s_0\}$

for every  $s \in Frontier \setminus S_g$  do

$Frontier \leftarrow Frontier \setminus \{s\}$

if  $Applicable(s) = \emptyset$  then return failure

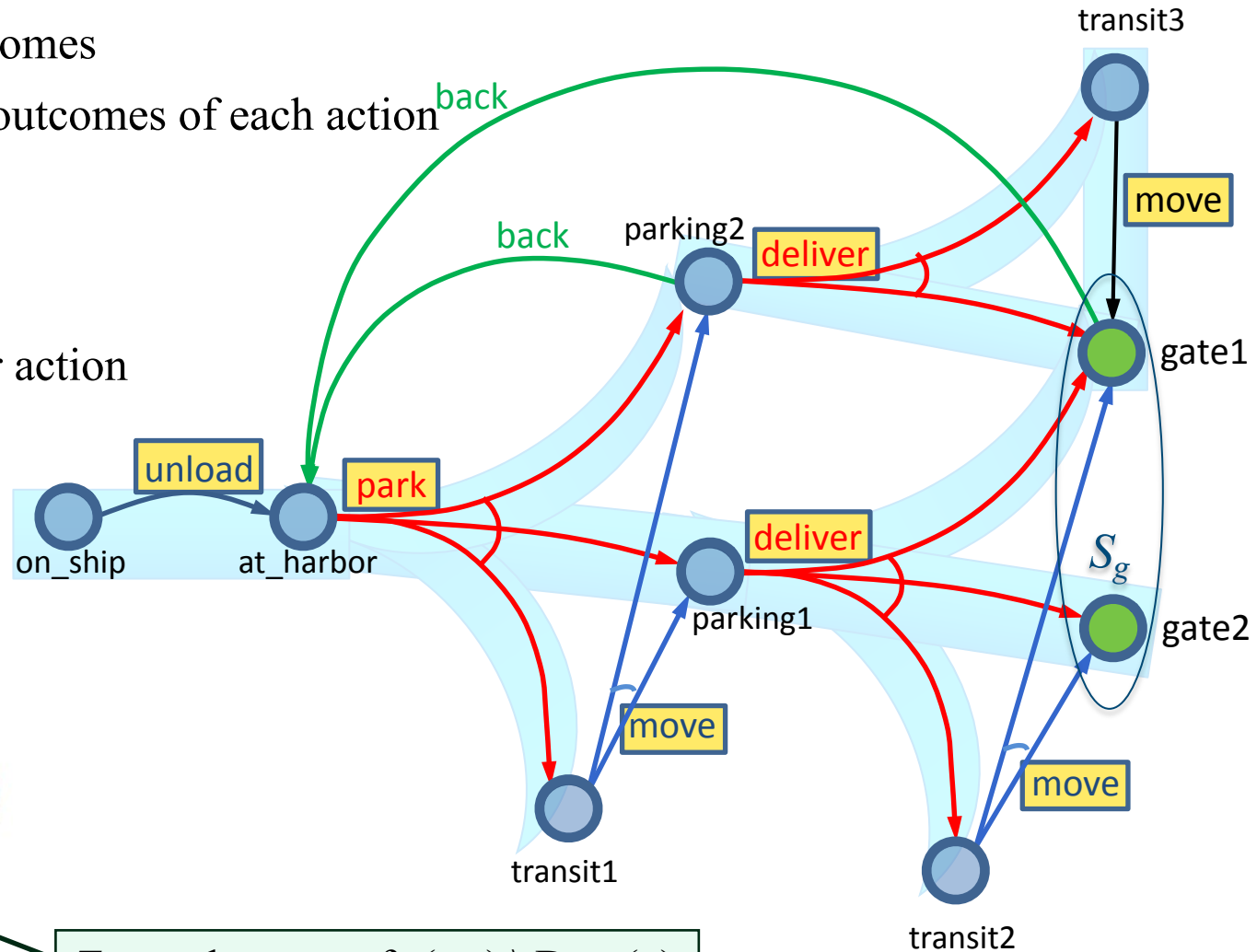
nondeterministically choose  $a \in Applicable(s)$

$\pi \leftarrow \pi \cup (s, a)$

~~$Frontier \leftarrow Frontier \cup (\gamma(s, a) \setminus Dom(\pi))$~~

if  $has-loops(\pi, a, Frontier)$  then return failure

return  $\pi$



$T \leftarrow i$  elements of  $\gamma(s, a) \setminus Dom(\pi)$   
 $Frontier \leftarrow Frontier \cup T$



# Safely Explorable Domains

- *Safely explorable* domain
  - ▶ for every state  $s$ , at least one goal state is reachable from  $s$
- For Lookahead, suppose we use Lookahead-Partial-Plan or FS-Replan
  - ▶ Then Lookahead never returns failure
- Every “fair” execution will eventually reach a goal

**Poll:** Suppose we just choose a random action each time. Is every “fair” execution guaranteed to reach a goal?

# Min-Max LRTA\*

```
Min-Max LRTA* ( $\Sigma, s_0, S_g$ )  
   $s \leftarrow s_0$   
  while  $s \notin S_g$  and  $\text{Applicable}(s) \neq \emptyset$  do  
     $a \leftarrow \operatorname{argmin}_{a \in \text{Applicable}(s)} \max_{s' \in \gamma(s,a)} h(s')$   
     $h(s) \leftarrow \max\{h(s), 1 + \max_{s' \in \gamma(s,a)} h(s')\}$   
    perform action  $a$   
     $s \leftarrow$  the current state
```

If some actions have cost  $\neq 1$ ,  
then use  $\text{cost}(s,a)$  here

- loop
  - ▶ choose an action  $a$  that (according to  $h$ ) has optimal worst-case cost
    - Update  $h(s)$  to use  $a$ 's worst-case cost
    - Perform  $a$
- In safely explorable domains with no “unfair” executions, guaranteed to reach a goal

# Example

Min-Max LRTA\* ( $\Sigma, s_0, S_g$ )

$s \leftarrow s_0$

while  $s \notin S_g$  and  $\text{Applicable}(s) \neq \emptyset$  do

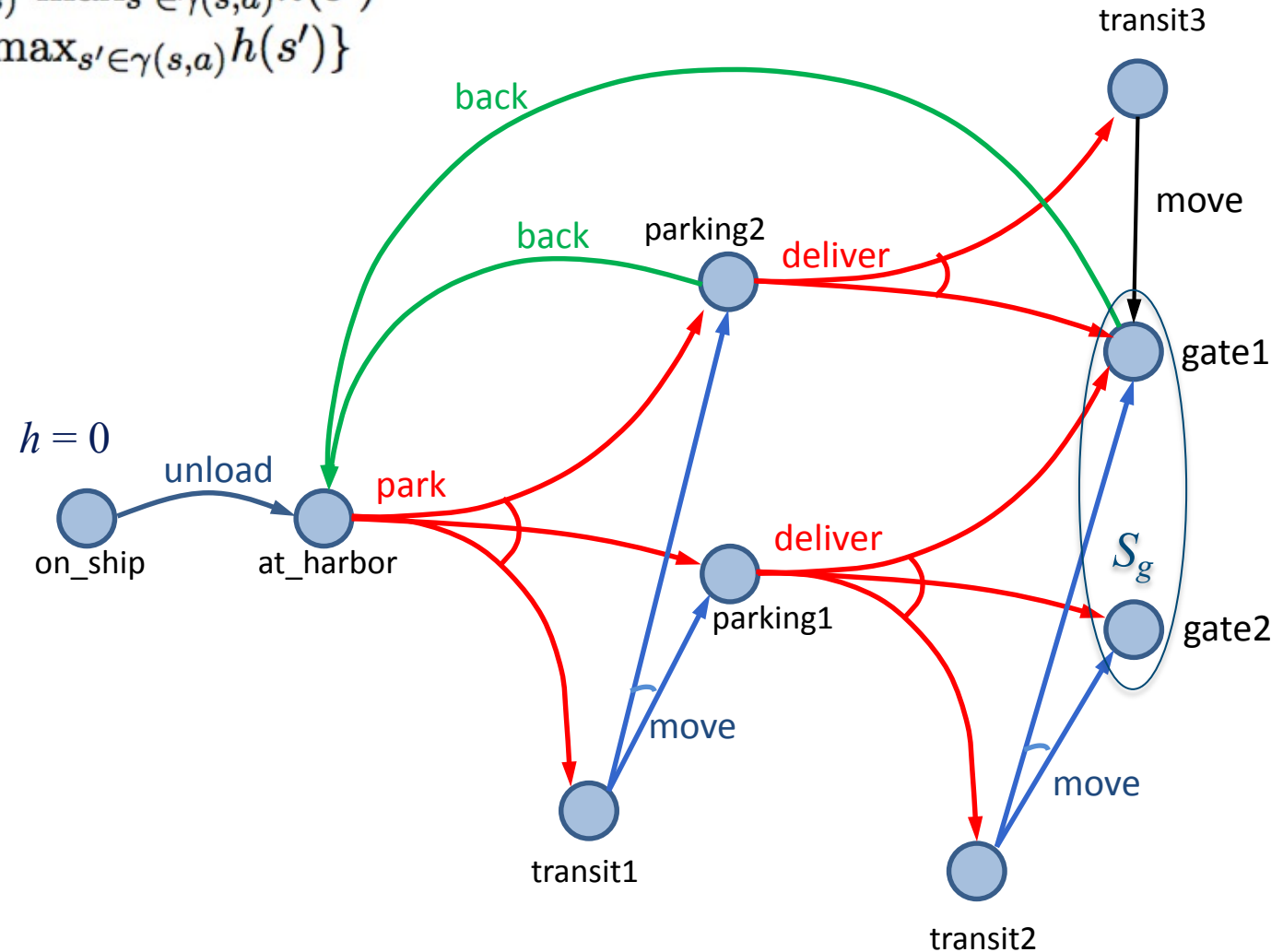
$a \leftarrow \operatorname{argmin}_{a \in \text{Applicable}(s)} \max_{s' \in \gamma(s,a)} h(s')$

$h(s) \leftarrow \max\{h(s), 1 + \max_{s' \in \gamma(s,a)} h(s')\}$

perform action  $a$

$s \leftarrow$  the current state

- Suppose that initially,  $h(s) = 0$  for every state  $s$



# Example

Min-Max LRTA\* ( $\Sigma, s_0, S_g$ )

$s \leftarrow s_0$

while  $s \notin S_g$  and  $\text{Applicable}(s) \neq \emptyset$  do

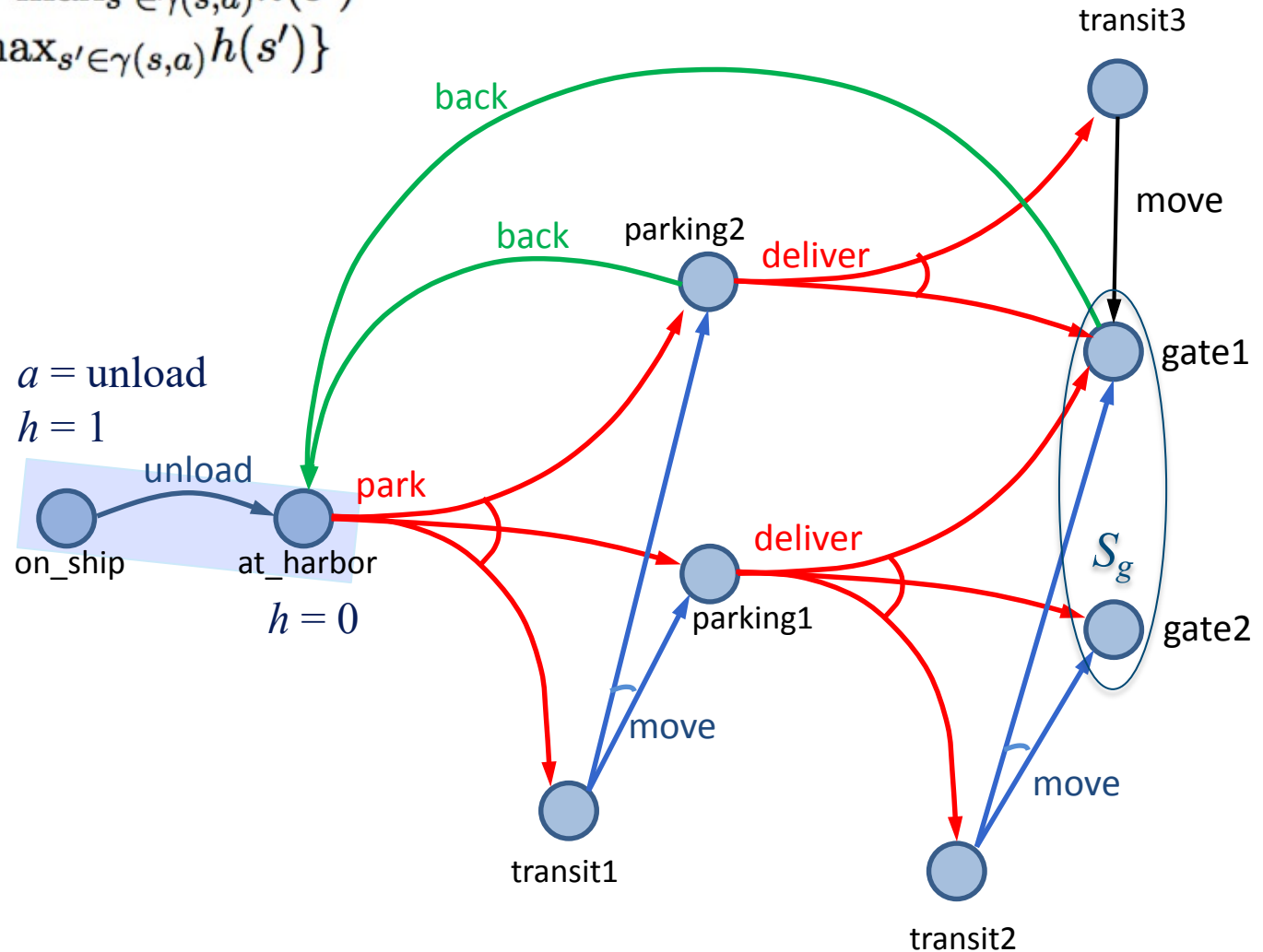
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perform action  $a$

$s \leftarrow$  the current state

- Suppose that initially,  $h(s) = 0$  for every state  $s$



# Example

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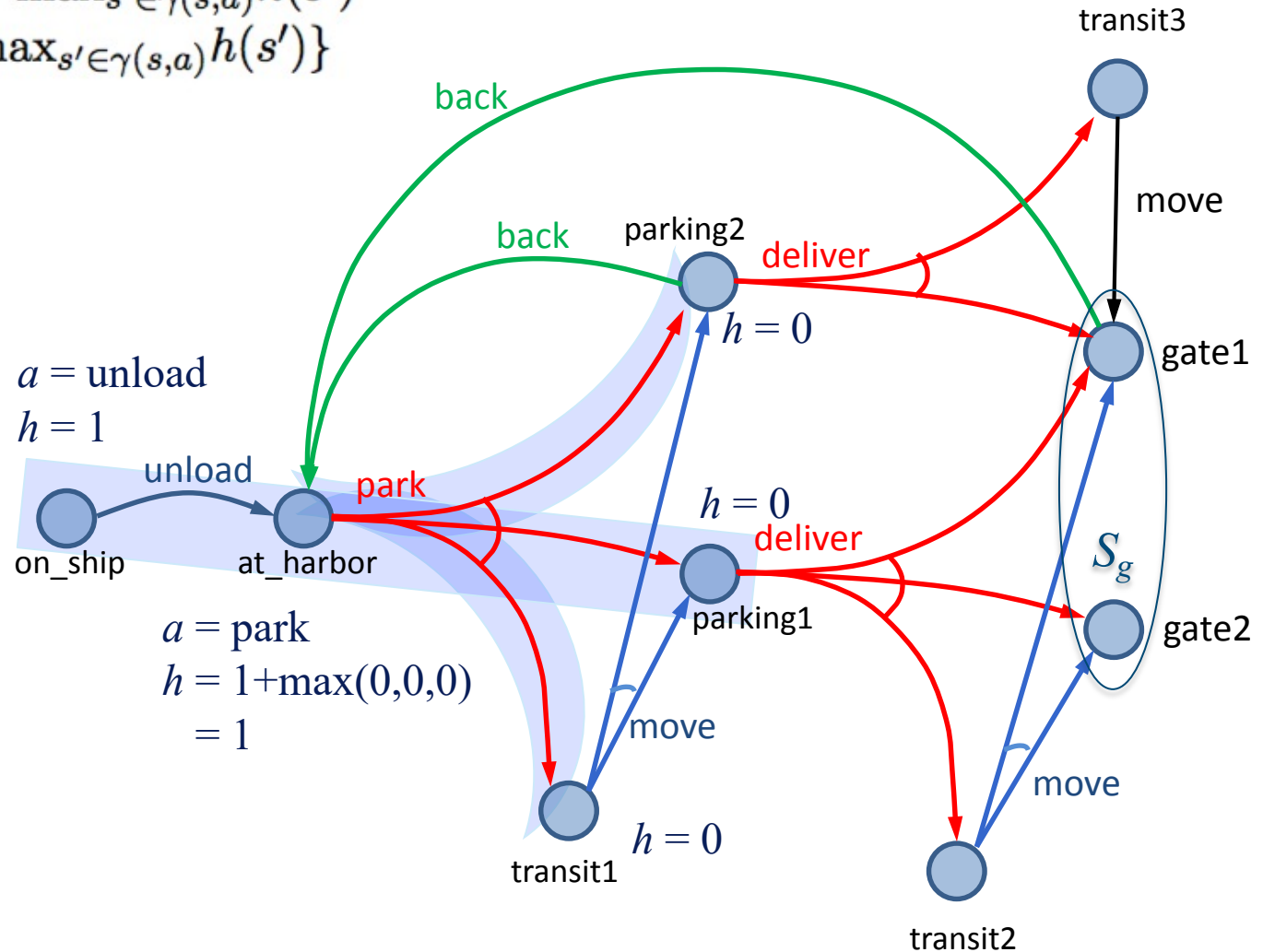
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perform action  $a$

$s \leftarrow$  the current state





# Example

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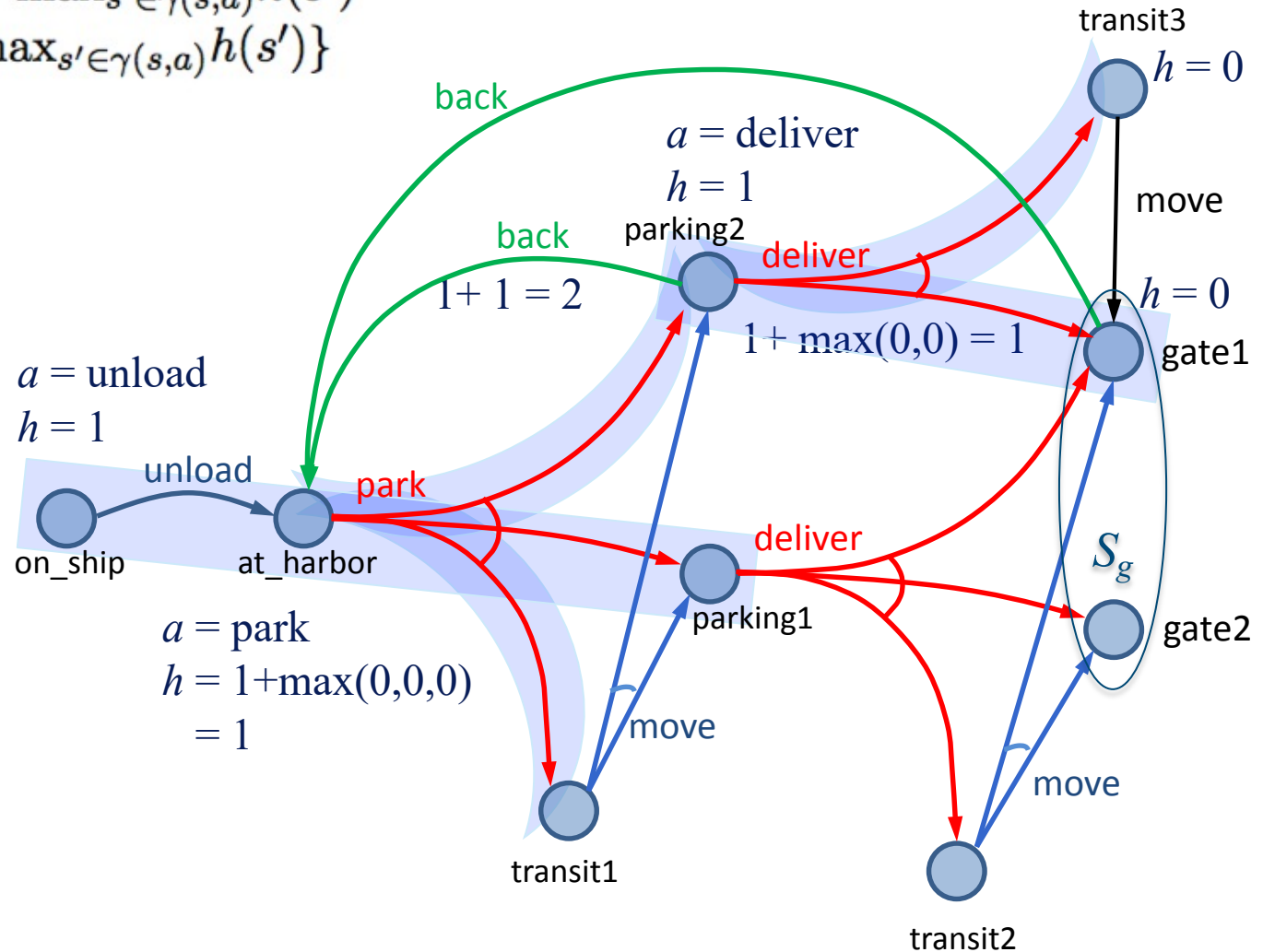
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$h(s) \leftarrow \max\{h(s), 1 + \max_{s' \in \gamma(s,a)} h(s')\}$

perform action  $a$

$s \leftarrow$  the current state



## 5.7 Refinement Methods

- Differences to refinement methods in Chapter 3:
  - ▶ Tasks refine into automata
  - ▶ Need to combine the automata
- Important work, but the concepts are complicated
  - ▶ We won't have time to cover them

# Summary

- Actions, plans, policies, planning problems
- types of solutions: unsafe, safe (acyclic, cyclic)
  - ▶ Find-solution, Find-acyclic-solution, Find-safe-solution
- Guided-find-safe-solution
  - ▶ call find-solution to get an unsafe solution
  - ▶ call find-solution again on the leaves
  - ▶ if dead-ends are encountered, modify actions that lead to them
- Find-safe-solution-by-determinization
  - ▶ Like Guided-find-safe-solution, but call classical planner on determinized domain, convert plan into policy
- Online approaches
  - ▶ Lookahead-partial-plan
    - adaptation of Run-Lazy-Lookahead
  - ▶ FS-replan
    - adaptation of Run-Lookahead
- Ways to do the lookahead
  - ▶ full breadth with limited depth,
    - iterative deepening
  - ▶ full depth with limited breadth
    - iterative broadening
  - ▶ convergence in safely explorable domains