AMSC 660 – Scientific Computing I

Prof. O'Leary

Term Project – The Assignment

"Modeling of a Seat Suspension System"

Gregory J. Hiemenz
Dec. 13, 2006

Figure 1 depicts a lumped parameter system that may be used to model the response of a seated human body in a seat suspension system [1,2,3]. In this model, the seat, denoted by mass M_1, is fixed to the floor through a damper (or energy absorber) that provides a force, F_{MR}, and through a spring, K_1. In addition, an end-stop buffer is implemented, which produces a nonlinear spring reaction force, F_{st}, when the suspension stroke exceeds its free-suspension travel. The soft seat cushion is simply represented as a stiffness and damping (K_{2c} and C_{2c}, respectively). This lumped parameter model assumes the human is seated and that 29% of the body weight is supported by the feet [1, 2]. The body is divided into four parts: pelvis, upper torso, viscera, and head, represented by mass M_i, stiffness K_i, and damping C_i, where i = 2, 3, 4, and 5, respectively. The displacement of the floor is given by z_0, and z_1 through z_5 are the absolute displacements of masses 1-5, respectively.

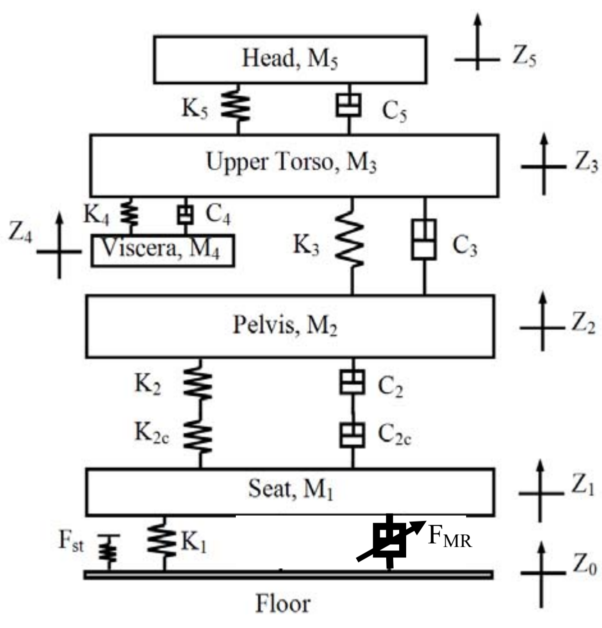

Figure 1 – Mechanical Model of Seat Suspension System Coupled with a Human Body in Seated Posture

The equations of motion for this system are obtained by summing the inertial, stiffness, and damping related vertical forces on each mass. The inertial force acting on a given mass is given by the mass times absolute acceleration ($M_i \ddot{z}_i$ - the dot denoting derivative with respect to time). The stiffness/spring force is given by the stiffness times the relative displacement between the two masses between which the springs are connected ($K_i(z_{i+1} - z_i)$). Similarly, the damping force is given by the damping coefficient times the relative velocity between the two masses between which the springs are connected ($C_i(\dot{z}_{i+1} - \dot{z}_i)$). Lastly, gravitational force on each mass must be included, $M_i g$, where g is the gravitational acceleration (9.81 m/s^2). The motion of these masses is then given by the following system of differential equations [1]:

$$M_1 \ddot{z}_1 = -K_1(z_1 - z_0) + K_{2t}(z_2 - z_1) + C_{2t}(\dot{z}_2 - \dot{z}_1) + F_{MR} + F_{st} - M_1 g \tag{1}$$

$$M_2 \ddot{z}_2 = -K_{2t}(z_2 - z_1) - C_{2t}(\dot{z}_2 - \dot{z}_1) + K_3(z_3 - z_2) + C_3(\dot{z}_3 - \dot{z}_2) - M_2 g \tag{2}$$

$$\begin{aligned} M_3 \ddot{z}_3 = &-K_3(z_3 - z_2) - C_3(\dot{z}_3 - \dot{z}_2) - K_4(z_3 - z_4) - C_4(\dot{z}_3 - \dot{z}_4) \\ &+ K_5(z_5 - z_3) + C_5(\dot{z}_5 - \dot{z}_3) - M_3 g \end{aligned} \tag{3}$$

$$M_4 \ddot{z}_4 = K_4(z_3 - z_4) + C_4(\dot{z}_3 - \dot{z}_4) - M_4 g \tag{4}$$

$$M_5 \ddot{z}_5 = -K_5(z_5 - z_3) - C_5(\dot{z}_5 - \dot{z}_3) - M_5 g\ , \tag{5}$$

where,

$$K_{2t} = \frac{K_2 K_{2c}}{K_2 + K_{2c}}, \text{ and } C_{2t} = \frac{C_2 C_{2c}}{C_2 + C_{2c}} \qquad (6, 7).$$

Problem 1:
Arrange this set of ODEs into standard form. In doing so, simplify to matrix form and identify all matrices used.

Now, to complicate matters, the cushion and biodynamic stiffnesses tend to be nonlinear. The cushion stiffness is given by:

$$K_{2c} = 3380.65\frac{e^{23.622(z_1-z_2)}-1}{z_1-z_2}. \tag{8}$$

The stiffness of the pelvis, K_2, is modeled by the nonlinear function [3]:

$$K_2 = \begin{cases} 8.1075\text{e}7(z_1-z_2)^2, & \text{if } (z_1-z_2)\ge 0 \\ 0, & \text{if } (z_1-z_2)<0 \end{cases} \tag{9}$$

The stiffness of the upper torso is also nonlinear [3]:

$$K_3 = \begin{cases} 3.78\text{e}6+1.09\text{e}7(z_2-z_3)-2.69\text{e}7(z_2-z_3)^2, \text{if}(z_2-z_3)\ge 0.04 \\ 77044, \quad \text{if}(z_2-z_3)<0.04 \end{cases} \tag{10}$$

The damping coefficient C_i is given by

$$C_i = 2\zeta_i\sqrt{M_iK_i} \qquad \text{if} \quad i=2,3,4,5 \tag{11}$$

where ζ_i is the damping ratio of each part of the human body. Because K_2 and K_3 are nonlinear functions, C_2 and C_3 are also nonlinear. Lastly, the nonlinear spring reaction force, F_{st}, due to the end-stop buffer is given by [1]:

$$F_{st} = \begin{cases} 0, & \text{if}\left|z_0-z_1\right|<0.025 \\ 8.0\text{e}4[z_0-z_1-z_{st}\,\text{sgn}(z_0-z_1)]+3.4\text{e}8[z_0-z_1-z_{st}\,\text{sgn}(z_0-z_1)]^3, & \text{if } \left|z_0-z_1\right|\ge 0.025 \end{cases} \tag{12}$$

Finally, the damper should be modeled using a Bingham-Plastic force model, which includes a viscous component and a friction component:

$$F_{MR} = C_1(\dot{z}_0-\dot{z}_1)+F_f\cdot\text{sgn}(\dot{z}_0-\dot{z}_1)\ , \tag{13}$$

where C_1 is the post-yield viscous damping coefficient, F_f is the friction force, and "sgn" represents the Signum function.

Problem 2:

a) *Using the parameters listed in Table 1 [1, 2, 3], write a Matlab function xdot=seat_system_ode(t,x) representing this system of nonlinear ODEs. Pass other necessary parameters through as global variables.*

b) *Solve this system using ode45 for 20 cycles of a 0.2g amplitude sinusoid floor acceleration. Record the time to complete this solution and plot the relative displacement & velocity between the seat and the floor and the absolute pelvis & head accelerations vs. time.*

   Note: The initial velocities are all zero. The initial displacements are the static 1g displacements (i.e., $x_1(0) = (M_1 + M_2 + M_3 + M_4 + M_5)\cdot g / K_1$). Assume only the initial cushion stiffness (K_{2c} = 37.7e3) for K_{2t} (i.e. $x_2(0) = (M_2 + M_3 + M_4 + M_5)\cdot g / K_{2c}$). Also, for simplicity assume $x_3(0)=x_4(0)=x_5(0)=x_2(0)$.

c) *Repeat part b using ode23, ode113, ode15s, ode23s, ode23t, &ode23tb. Use default options for each case. Explain why some solution methods fail and/or have longer solution times than others.*

Table 1 - Seat Suspension Model Parameters

Quantity	**Symbol**	**Value**	**Units**
Mass of seat	M_1	11.5	kg
Mass of pelvis	M_2	29	kg
Mass of upper torso	M_3	21.8	kg
Mass of viscera	M_4	6.8	kg
Mass of head	M_5	5.5	kg
Stiffness of coil spring	K_1	50.0	kN/m
Stiffness of viscera	K_4	2.84	kN/m
Stiffness of head	K_5	202.3	kN/m
Post-Yield Damping Coefficient	C_1	750	N·s/m
Cushion Damping	C_{2c}	159	N·s/m
Pelvis Damping	ζ_2	0.25	-
Torso Damping	ζ_3	0.11	-
Viscera Damping	ζ_4	0.5	-
Head Damping	ζ_5	0.1	-
Damper Friction Force	F_f	75	N

The Bingham-plastic force model (Eq. 13) for the damper may be approximated using a hypertangent function below as depicted in Figure 2:

$$F_{MR} = C_1\left(\dot{z}_1 - \dot{z}_o\right) + F_f \cdot \tanh\left(\frac{\dot{z}_1 - \dot{z}_o}{\varepsilon}\right), \tag{14}$$

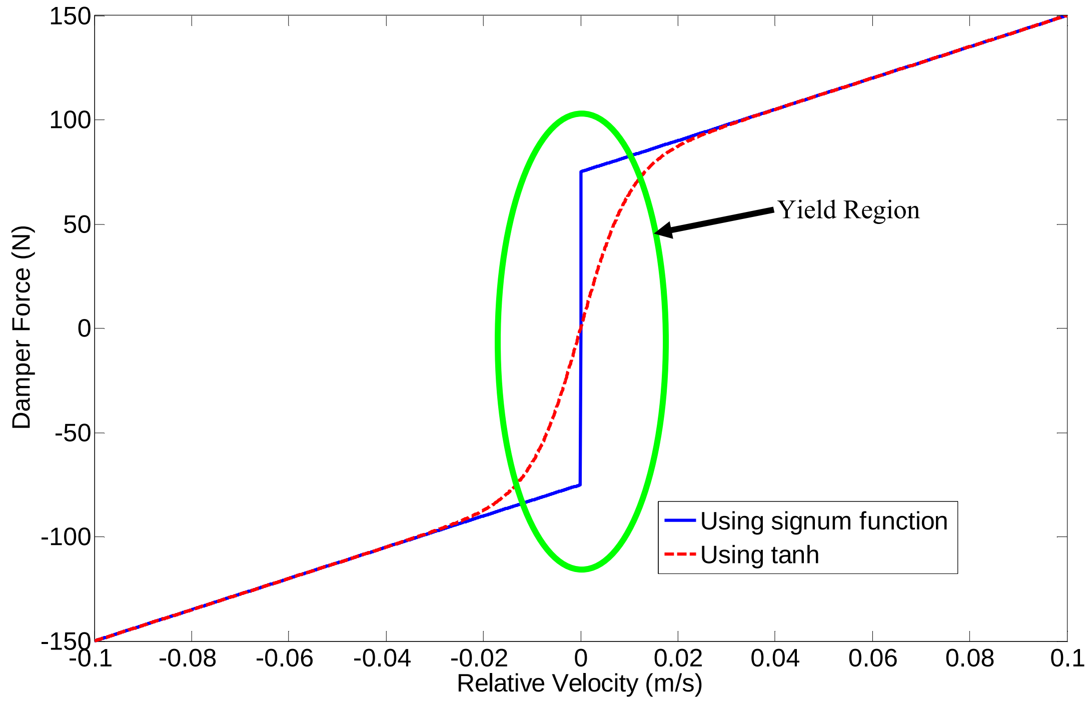

Figure 2 - Approximating the Bingham-Plastic Force Model

<u>Problem 3:</u>

a) Repeat Problem 2b & 2c using this hypertangent model for the damper with ε=0.005. How have the results changed? Why? Do you recommend this approximation?

b) Repeat with a 20g amplitude floor acceleration. How have the time results changed? Why?

Problem 4:

a) Write a Matlab function [t,x]=rk(f,T,X0) to perform a fixed step integration using 4^{th} order Runge-Kutta algorithm. Here, f is the ode function, T is a time vector, and X0 are the initial conditions.
b) Rerun problem 3a using rk instead of ode45. How do the resulting plots and solution time compare with Matlab's ODE solvers? Give a benefit and a pitfall to using this routine over Matlab's ODE solvers.

References

1. Choi, Y.T. and Wereley, N.M., Biodynamic response mitigation to shock loads using magnetorheological helicopter crew seat suspensions, Journal of Aircraft, Vol. 42, No. 5, 2005, pp. 1288-1295.
2. Zong, Z. and Lam, K.Y., Biodynamic response of shipboard sitting subject to ship shock motion, Journal of Biomechanics, Vol. 35, 2002, pp. 35-43.
3. Liu, X.X., Shi, J., Li, G.H., et al., Biodynamic response and injury estimation of ship personnel to ship shock motion induced by underwater explosion, Proceedings of the 69th Shock and Vibration Symposium, St. Paul, Vol. 18, 1998, pp. 1-18.