1. Suppose that we have \( n \) uniformly distributed random vectors \( x_j \in S \), where
\[
S = \{ x \in \mathbb{R}^5 : -2 \leq x_i \leq 2, \ i = 1, \ldots, 5 \}.
\]
- Suppose we have a region \( T \subset S \) and a function \( \hat{T}(x) \) that equals 1 if \( x \in T \) and 0 otherwise.

(a) Use this information to estimate the volume of \( T \).
(b) In what way would your estimate improve if you were given \( 2n \) points?

2. Consider the following counting problem. Let \( C(k) \) be the number of distinct arrangements of \( k \) dimers on a particular lattice. We are told that \( C(20) = 200 \), and we want to determine \( C(55) \).

Suppose that we have sampled 6,000,000 arrangements (uniformly among all possible arrangements), counted the number of dimers in each of these arrangements, and stored the counts in a 6,000,000×1 vector called \( d \). Write MATLAB statements to estimate \( C(55) \).