AMSC 614 - SPRING 08

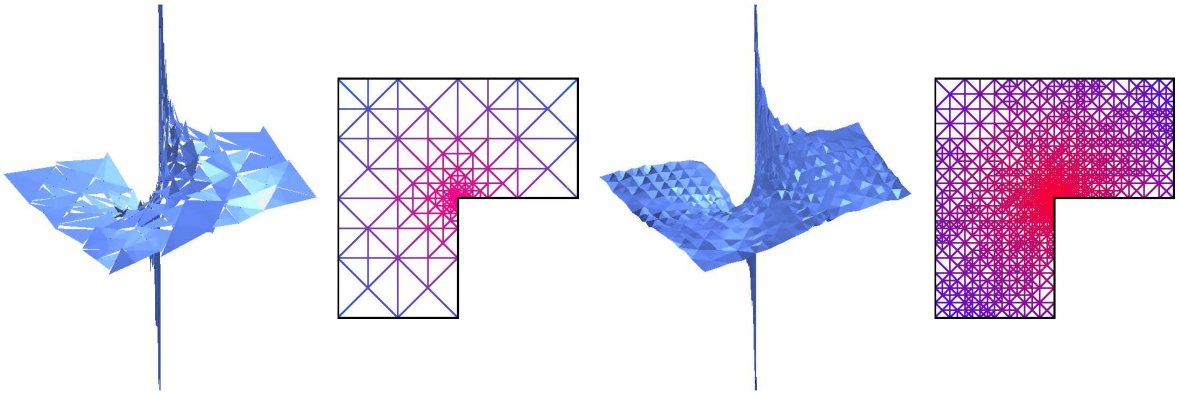
MATHEMATICS OF THE FINITE ELEMENT METHOD

Instructor: RICARDO H. NOCHETTO

Time and Place: Tu-Th 9:30-10:45 MTH 0401

Webpage: `www.math.umd.edu/~rhn/teaching.html`

Objectives. The finite element method (FEM) is one of the most successful computational tools in dealing with partial differential equations (PDE) arising in science and engineering (solid and fluid mechanics, electromagnetism, thermodynamics, etc). The formulation of FEM, their properties, stability, convergence, and fast solvers (multilevel methods and preconditioners) will be discussed. Their actual implementation will also be addressed mainly via MATLAB computer projects.



Stokes flow over an L-shaped domain: Pressures and meshes for error tolerance of 5% and unstable finite element pairs (resp. DOFs): $\mathcal{P}^2$-$\mathcal{P}^1_d$ (1940), $\mathcal{P}^1$-$\mathcal{P}^1$ (4971). The oscillations do not persist under further selective refinement (nonlinear stabilization effect of adaptivity). Error estimation and adaptivity will be fully discussed in this course.

Prerequisities. Some basic functional analysis and PDE theory (variational method) will be reviewed. Therefore prior exposure to graduate level PDE and MATLAB will be useful but not mandatory. This course may be an excellent complement to the graduate PDE course MATH 674, which covers Sobolev spaces and modern PDE theory.

Text: D. Braess, Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics, Cambridge Univ. Press, 2nd ed (2001), ISBN 0521 01195 7.

Syllabus

- Variational Formulation of Elliptic Problems and Examples.
- The Finite Element Method and its Implementation.
- Polynomial Interpolation Theory in Sobolev Spaces.
- A Priori Error Estimates and Applications.
- A Posteriori Error Estimates and Adaptivity.
- Fast Solvers: Multigrid Methods and Multilevel Preconditioners.
- Variational Crimes: Nonconformity, Quadrature, Isoparametric Finite Elements.
- Mixed FEM: inf-sup condition and stable spaces, applications to Stokes Flow.

Evaluation: Homeworks, both theoretical and computational. Basic MATLAB programs will be distributed and will have to be modified appropriately.