1. (3) At what rate does the sequence

\[ e_k = 1 + (0.5)^{2^k} \]

converge to 1?

2. (5) Let \( x = Sz + d \), where \( S \) is a given \( n \times n \) matrix and \( d \) is a given \( n \times 1 \) vector.

Suppose \( f \) is a function of \( n \) variables, and define \( \hat{f}(z) = f(x) = f(Sz + d) \).

Write expressions for the gradient and Hessian of \( \hat{f} \) with respect to the variables \( z \), using the gradient and Hessian of the function \( f \). (Hint: compute \( \partial \hat{f}/\partial z_j \) by using the chain rule and the values \( \partial f/\partial x_i \) and \( \partial x_i/\partial z_j \).)

3. Consider the following problem: Find a value of \( \gamma \) so that the solution \( p \) to the linear system

\[ (H + \gamma I)p = -g \]

satisfies \( \|p\|_2 = \delta \), where \( \delta > 0 \) is a given value.

Suppose we have factored \( H = U \Lambda U^T \), where \( \Lambda \) is a diagonal matrix with diagonal elements \( \lambda_i \) and \( U \) is orthogonal, so that \( UU^T = U^T U = I \).

3a. (5) Write the solution \( p \) to the linear system in terms of \( g \), \( \gamma \), \( U \), \( u_i \), and \( \lambda_i \), where \( u_i \) is the \( i \)th column of \( U \). (Hint: Remember that scalars like \( \gamma \) commute with matrices.)

3b. (2) Show that, for any vector \( w \), \( \|w\|_2 = \|U^T w\|_2 \).

3b. (5) Use 3b to write an expression for \( \|p\|_2 \) in terms of \( g \), \( \gamma \), \( U \), and \( \lambda_i \).

3c. (5) Find an interval for \( \gamma \) on which \( \|p\|_2 \) is monotonically decreasing. (Hint: Remember that some of the \( \lambda_i \) might be negative.)

3d. (5) Describe how you could use MATLAB’s \texttt{fzero} to find a value of \( \gamma \) for which \( \|p\|_2 = \delta \) (if such a value exists). What initial interval would you give \texttt{zeroin}?

In Homework 2, you will write a program, using this algorithm to solve minimization problems.