AMSC 607 / CMSC 764 Homework 3, Fall 2010
20 points
Due September 28, before class begins.

In class, we noted that any symmetric matrix $\boldsymbol{A}$ has an eigendecomposition $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^T$, where $\boldsymbol{\Lambda}$ is a diagonal matrix containing the eigenvalues and $\boldsymbol{U}$ has the eigenvectors as its columns. Since the eigenvectors are orthogonal, $\boldsymbol{U}\boldsymbol{U}^T = \boldsymbol{U}^T\boldsymbol{U} = \boldsymbol{I}$.

4.

(a) (5) Consider the trust region step, the solution to

$$\min_{\boldsymbol{p}} f(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})^T\boldsymbol{p} + \frac{1}{2}\boldsymbol{p}^T\boldsymbol{H}(\boldsymbol{x})\boldsymbol{p}$$

subject to

$$\boldsymbol{p}^T\boldsymbol{p} \leq \delta$$

where δ is a given number. Determine the values of α_i so that

$$\boldsymbol{p} = \sum_{i=1}^{n} \alpha_i \boldsymbol{u}_i\,.$$

(Recall that another way to define the solution is

$$(\boldsymbol{H}(\boldsymbol{x}) + \gamma\boldsymbol{I})\boldsymbol{p} = -\boldsymbol{g}(\boldsymbol{x}),$$

where γ is the Lagrange multiplier for the problem.)

(b) (5) Use your expression from (a) to show that $\boldsymbol{p}^T\boldsymbol{p}$ is a monotonically decreasing function of the Lagrange multiplier, and that the Lagrange multiplier is a nonincreasing function of δ.

(c) (5) Use your expression from (a) to determine the limit of the direction of $\boldsymbol{p}$ as $\delta \to 0$ and the limit as $\delta \to \infty$.

(d) (5) Show that

$$\boldsymbol{B}_{k+1} = \boldsymbol{B}_k + \frac{(\boldsymbol{y}_k - \boldsymbol{B}_k\boldsymbol{s}_k)(\boldsymbol{y}_k - \boldsymbol{B}_k\boldsymbol{s}_k)^T}{(\boldsymbol{y}_k - \boldsymbol{B}_k\boldsymbol{s}_k)^T\boldsymbol{s}_k}$$

satisfies the secant condition and is symmetric if $\boldsymbol{B}_k$ is.
