AMSC 607 / CMSC 764 Homework 4, Fall 2010
20 points
Due October 5, before class begins.

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5. Let

$$f(\boldsymbol{x}) = 3x_1^2 + x_2^2 - 2x_1x_2 + x_1^3 + 2x_1^4,$$

and let $\boldsymbol{x}^{(0)} = [0, -2]^T$.

You might want to use MATLAB to do the calculations in this problem, but it is not required.

(a) (5) Is $\boldsymbol{p}^{(0)} = [0, 1]^T$ a descent direction for f at $\boldsymbol{x}^{(0)}$?

(b) (7) Find the range of values α for which the step $\boldsymbol{x}^{(1)} = \boldsymbol{x}^{(0)} + \alpha\boldsymbol{p}^{(0)}$ satisfies the Goldstein conditions (see "Basics" notes, p. 20) for $\rho = .25$. (It is ok to solve this graphically, using the magnifying glass on a MATLAB figure so that you can read α values to about 2 significant digits.)

(c) (8) Compute $\boldsymbol{x}^{(1)}$ and $\boldsymbol{x}^{(2)}$ for the Fletcher-Reeves nonlinear conjugate gradient algorithm started with $\boldsymbol{x}^{(0)}$, using a stepsize of $\alpha = 0.25$.

6.

(a) (10) Write a MATLAB function to apply the limited memory quasi-Newton method, with `cvsrch.m` for the line search, to minimize an arbitrary function $f(\boldsymbol{x})$. The calling sequence should be `x = lmmin(@f,x0,delta,maxit,nupd)` where `@f` is a user-supplied function to evaluate f at a given value $\boldsymbol{x}$ and `x0` is the starting guess. The parameter `nupd` specifies how many DFP updates to save. (For simplicity, you can store and update the $\boldsymbol{C}$ matrix. Note in your documentation that this is not the recommended way to implement the algorithm and explain what is recommended.) Stop the iteration when $\|\boldsymbol{g}(\boldsymbol{x}^{(k)})\| \le \delta\|\boldsymbol{g}(\boldsymbol{x}^{(0)})\|$, or after `maxit` iterations, whichever comes first. Document your function well.

(b) (10) Try your program on the n-dimensional Rosenbrock banana function,

$$f(\boldsymbol{x}) = \sum_{i=1}^{n-1} (1 - x_i)^2 + 100(x_{i+1} - x_i^2)^2,$$

with $n = 50$, starting at $\boldsymbol{x} = [-1, -2, -2, \ldots, -2]^T$ with `delta = 1.e-5`. and `maxit = 500`. Use $\boldsymbol{C}^{(0)} = \boldsymbol{I}$, `nupd = 1, 2, 4`, and discuss the results.
