Consider Problem A:

$$\min_{\mathbf{x}} \mathbf{x}^T Q_0 \mathbf{x} + p_0^T \mathbf{x} + r_0$$

subject to

$$\mathbf{x}^T Q_1 \mathbf{x} + p_1^T \mathbf{x} + r_1 \leq 0,$$
$$\mathbf{x}^T Q_2 \mathbf{x} + p_2^T \mathbf{x} + r_2 \leq 0,$$

where $\mathbf{x}$ is $n \times 1$ and $Q_i$ is $n \times n$ and positive semidefinite, for $i = 0, 1, 2$,

12a. (6) Show, directly from the definition of a convex set, that the set of $\mathbf{x}$ that are feasible for Problem A is convex. (Therefore, this is a convex programming problem.)

12b. (7) Write Problem A as a semi-definite programming problem (SDP) in primal form (p. 18 of the notes). In other words, define $C, X, A(X)$, and $b$. Hint: Use the eigendecomposition of $Q_i$ to write $Q_i = B^T B$ for some matrix $B$. (This is problem 8.4, p. 655, from Griva, Nash, and Sofer.)

12c. (7) Prove that if $X$ is primal feasible for an SDP and $(y, S)$ are dual feasible, then $C \bullet X - b^T y = X \bullet S \geq 0$. (This is problem 8.10, p.656, from Griva, Nash, and Sofer.)
13. Consider Problem B:

\[
\min_{x} w^T x
\]

subject to

\[
\|Ax - b\|_2^2 \leq d,
\]

\[
x \geq 0.
\]

13a. (7) For \(n = 2\), sketch the feasible region and the level curves \(w^T x = \text{constant}\) and indicate where the solution point for Problem B is. (I’m not giving you a particular choice of data matrices and vectors, so you are just sketching a “generic” picture to get some intuition for the problem. Choose specific data if it helps you, but it is not required.)

13b. (7) Express Problem B as a SOCP (p. 17 of the notes). In other words, define \(f, A_i, b_i, c_i, d_i,\) and \(m\).

13c. (6) Suppose we do not have software to solve an SOCP. Explain how we can solve Problem B by using a zerofinder (i.e., a program to solve the single nonlinear equation \(F(c) = 0\)) and an algorithm to solve the quadratic programming problem

\[
\min_{x} \|Ax - b\|_2^2 - d
\]

subject to

\[
w^T x = c
\]

\[
x \geq 0.
\]