AMSC 607 / CMSC 764 Homework 9, Fall 2010
Due November 16, before class begins.

Note: Both of these are written problems; it's a busy time of the semester and I didn't want to burden you with programming. But if you would like to substitute one or two programming problems, contact me with a proposal.

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12. Consider Problem A:

$$\min_{\boldsymbol{x}} \boldsymbol{x}^T \boldsymbol{Q}_0 \boldsymbol{x} + \boldsymbol{p}_0^T \boldsymbol{x} + r_0$$

subject to

$$\begin{aligned} \boldsymbol{x}^T \boldsymbol{Q}_1 \boldsymbol{x} + \boldsymbol{p}_1^T \boldsymbol{x} + r_1 &\le 0, \\ \boldsymbol{x}^T \boldsymbol{Q}_2 \boldsymbol{x} + \boldsymbol{p}_2^T \boldsymbol{x} + r_2 &\le 0, \end{aligned}$$

where $\boldsymbol{x}$ is $n \times 1$ and $\boldsymbol{Q}_i$ is $n \times n$ and positive semidefinite, for $i = 0, 1, 2$,

12a. (6) Show, directly from the definition of a convex set, that the set of $\boldsymbol{x}$ that are feasible for Problem A is convex. (Therefore, this is a convex programming problem.)

12b. (7) Write Problem A as a semi-definite programming problem (SDP) in primal form (p. 18 of the notes). In other words, define $\boldsymbol{C}$, $\boldsymbol{X}$, $\boldsymbol{A}(\boldsymbol{X})$, and $\boldsymbol{b}$. Hint: Use the eigendecomposition of $\boldsymbol{Q}_i$ to write $\boldsymbol{Q}_i = \boldsymbol{B}^T \boldsymbol{B}$ for some matrix $\boldsymbol{B}$. (This is problem 8.4, p. 655, from Griva, Nash, and Sofer.)

12c. (7) Prove that if $\boldsymbol{X}$ is primal feasible for an SDP and $(\boldsymbol{y}, \boldsymbol{S})$ are dual feasible, then $\boldsymbol{C} \bullet \boldsymbol{X} - \boldsymbol{b}^T \boldsymbol{y} = \boldsymbol{X} \bullet \boldsymbol{S} \ge 0$. (This is problem 8.10, p.656, from Griva, Nash, and Sofer.)

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13. Consider Problem B:

$$\min_{\boldsymbol{x}} \boldsymbol{w}^T \boldsymbol{x}$$

subject to

$$\|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}\|_2^2 \le d,$$

$$\boldsymbol{x} \ge \boldsymbol{0}.$$

13a. (7) For $n = 2$, sketch the feasible region and the level curves $\boldsymbol{w}^T \boldsymbol{x} =$ constant and indicate where the solution point for Problem B is. (I'm not giving you a particular choice of data matrices and vectors, so you are just sketching a "generic" picture to get some intuition for the problem. Choose specific data if it helps you, but it is not required.)

13b. (7) Express Problem B as a SOCP (p. 17 of the notes). In other words, define $\boldsymbol{f}$, $\boldsymbol{A}_i$, $\boldsymbol{b}_i$, $\boldsymbol{c}_i$, d_i, and m.

13c. (6) Suppose we do not have software to solve an SOCP. Explain how we can solve Problem B by using a zerofinder (i.e., a program to solve the single nonlinear equation $F(c) = 0$) and an algorithm to solve the quadratic programming problem

$$\min_{\boldsymbol{x}} \|\boldsymbol{A}\boldsymbol{x} - \boldsymbol{b}\|_2^2 - d$$

subject to

$$\boldsymbol{w}^T \boldsymbol{x} = c$$

$$\boldsymbol{x} \ge \boldsymbol{0}.$$