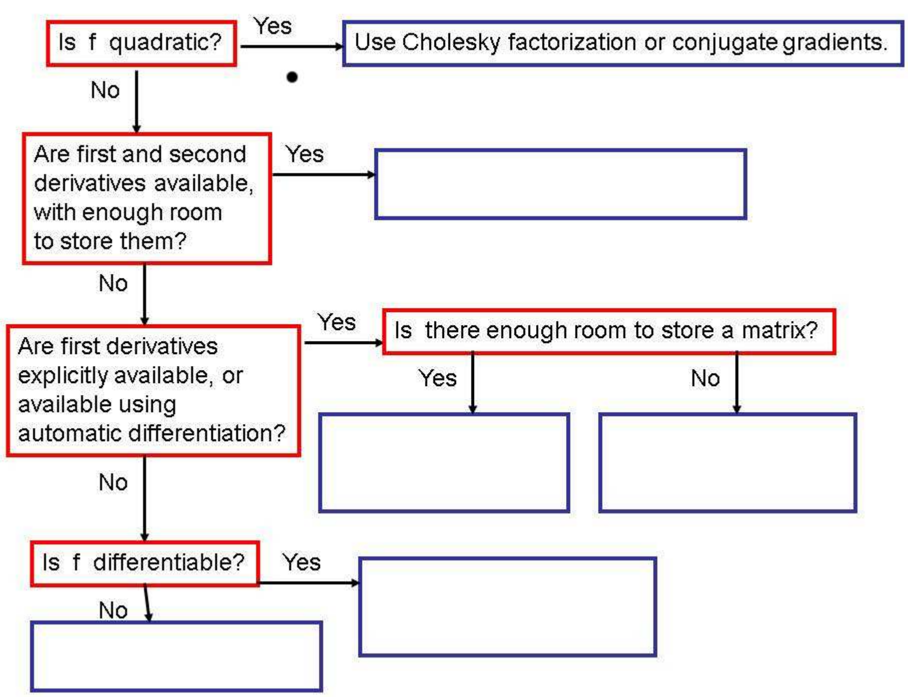
AMSC 607 /CMSC 764 Final Exam , Fall 2010

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. You may use any books and notes that you brought, but you many not share anything with others. No electronic devices are allowed.

Name ____

1. (20) Complete the following table of algorithms for minimizing a function $f(\boldsymbol{x})$, filling each empty box with the name of one good choice of algorithm (even if there are several good choices).



2. (20) Write the 1st order optimality conditions for the problem

$$\min_{\boldsymbol{x}} e^{(x_1-5)} + 6x_2^4 + x_1x_2 + x_3^2 + x_4^2 + 3x_2 + 5$$

subject to

$$\begin{aligned} \boldsymbol{Ax} &= \boldsymbol{b}, \\ x_1^2 + x_2^2 &\leq 1, \\ \boldsymbol{x} &\geq \boldsymbol{0}, \end{aligned}$$

where $\boldsymbol{A}$ and $\boldsymbol{b}$ are given and $\boldsymbol{x} \in \mathcal{R}^4$.

(In other words, explicitly write the necessary conditions for $\boldsymbol{x}_{opt}$ to satisfy to ensure that it is a local solution to this particular problem.)

3. (20) Reduce the system of equations

$$\begin{bmatrix} \boldsymbol{D} & \mathbf{0} & \boldsymbol{E} \\ \boldsymbol{A} & \boldsymbol{Q} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{B} & \boldsymbol{F} \end{bmatrix} \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{y} \\ \boldsymbol{z} \end{bmatrix} = \begin{bmatrix} \boldsymbol{a} \\ \boldsymbol{b} \\ \boldsymbol{c} \end{bmatrix}$$

to a linear system involving only the variables $\boldsymbol{y}$. In addition to writing down the final system, show how the system is derived and give formulas that can be used to compute $\boldsymbol{x}$ and $\boldsymbol{z}$ once $\boldsymbol{y}$ is known. Assume that $\boldsymbol{D}$, $\boldsymbol{E}$, and $\boldsymbol{F}$ are square and full rank.

4. (20) Write pseudo-code, at the level of detail of a MATLAB program, for a predictor-corrector primal-dual interior point method to solve the problem

$$\min_{\boldsymbol{x}} \boldsymbol{c}^T \boldsymbol{x}$$

subject to

$$\begin{aligned} \boldsymbol{A}\boldsymbol{x} &= \boldsymbol{b}, \\ \boldsymbol{x} &\geq \boldsymbol{0}, \end{aligned}$$

where $\boldsymbol{x}, \boldsymbol{c} \in \mathcal{R}^n$ and $\boldsymbol{b} \in \mathcal{R}^m$ with $m < n$. Assume a constraint qualification, and assume that the initial point $\boldsymbol{x}^{(0)}$ is feasible. Specify the inputs and the outputs and use the normal equations. Your method should be efficient in time and storage.

5. We considered Renegar's proof of polynomial complexity for a particular interior point method for solving linear programming problems.

5a. (8) Why is polynomial complexity important?

5b. (12) List three of the main ingredients in Renegar's proof, and explain in intuitive terms why they are important.

5c. (5) (Extra) What are the username and password for the "closed" part of the website? Why did I associate that password with that username?