This part of the quiz is take-home, due **when class begins** on November 1. Show all work. By putting your name on your submission, you agree to abide by the university’s code of academic integrity in completing the quiz. Use no communication with others, including classmates, tutors, etc..

Do Page 159, problem P4.4.3.

a. Show the derivation you used for the formula that you put into SplineQ. Also include a listing of your function.
b. Test your function on
\[ \int_a^b \sin x dx. \]
Use the knots \( x=\text{linspace}(0,\pi/2,m) \) and set \( a=.125, \ b=.45*\pi \). Make a table of the errors in the estimated integral for \( m = 2^k \), for \( k = 3, 4, \ldots, 10 \).
c. Determine the convergence rate for spline quadrature on this example. Hint: A method has a convergence rate of \( r \) if the error is approximately \( cm^r \) for some constant \( c \) when \( m \) is large. So use your computed errors to determine \( r \) experimentally.
Your submission should look like this:

- **Page 1:** a 3-column table, produced by Matlab, with each line giving \( k \), the estimated integral, the absolute error in the estimate, and (for \( k > 3 \)) the ratio of the error for \( k \) to the error for \( k - 1 \). Make sure that you format your table to make it easy to read and to give enough digits to be meaningful. (Hint: Use `fprintf`.)

- **Page 2:** Your derivation for part a, and your estimate of \( r \) and how you got it. (Use more than one page if necessary.)

- The remaining pages: a listing of the m-file containing your main program and a listing of your version of SplineQ. Make sure SplineQ is well-documented; review the answer sheet for Homework 1.