1. (10) Let

\[ f(x) = x^2 - 4. \]

Suppose our guess at the zero of this nonlinear equation is 2.5. If we apply Newton’s method, what would the next guess be?

**Answer:** We compute \( f'(x) = 2x \), so

\[
x \leftarrow x - \frac{x^2 - 4}{2x} = 2.5 - \frac{(2.5)^2 - 4}{5}.
\]

**Note:** This works out to be 2.05. Note that the guess 2.5 has one significant figure, and after 1 iteration we have 2: very fast convergence. This is the basis for the algorithm usually used in the `sqrt` function in languages like C++, Fortran, and Matlab. The initial guess is obtained by saling and then table look-up.

2. (10) Suppose we want to solve the problem

\[
\min_x \frac{1}{2} x_1^2 + \frac{3}{2} x_2^2 + x_1 - x_2 + 5
\]

and we have the initial guess \( x_1 = 2, x_2 = 3 \). What is the steepest descent direction from this guess?

**Answer:** The gradient of the function is

\[
\begin{bmatrix}
\frac{\partial f}{\partial x_1} \\
\frac{\partial f}{\partial x_2}
\end{bmatrix} = \begin{bmatrix}
x_1 + 1 \\
3x_2 - 1
\end{bmatrix} = \begin{bmatrix}
3 \\
8
\end{bmatrix},
\]

so the steepest descent direction is

\[- \begin{bmatrix}
3 \\
8
\end{bmatrix}.
\]