AMSC/CMSC 460 Quiz 8 , Fall 2001

1. (10) Let

$$\begin{aligned} y' &= y^2 - 5t \\ y(0) &= 1 \end{aligned}$$

Apply a PECE scheme to this problem, using Euler and Backward Euler with a stepsize $h = .1$, to obtain an approximation for $y(.1)$.

Answer:

P: $y(.1) = y(0) + h(1) = 1.1$
E: $f(.1, 1.1) = 1.21 - .5 = .71$
C: $y(.1) = y(0) + h(.71) = 1.071$ (answer)
E: $f(.1, 1.071) = (1.071)^2 - .5$

2. (10) Suppose we have two approximations to y_{n+1}

$$\begin{aligned} y_{n+1} &= y_n + \frac{h}{2}(3f_n - f_{n-1}) \\ y_{n+1} &= y_n + \frac{h}{12}(5f_{n+1} + 8f_n - f_{n-1}) \end{aligned}$$

the error formula for the first is $\frac{5h^3}{12}y^{(3)}(\eta)$ and for the second is $-\frac{h^4}{24}y^{(4)}(\eta)$. How would you estimate the error in the first approximation?

Answer: Since the 2nd formula is $O(h)$ more accurate than the first, we use the same reasoning as we did for the 'red Q' and 'black Q' in the numerical integration chapter and estimate the error in the first estimate for y_{n+1} by taking the second estimate minus the first.