AMSC/CMSC 460 Quiz 1 , Fall 2002

For this page of the quiz, assume you have a base 2 computer that stores floating point numbers using a 5 bit normalized mantissa (x.xxxx), a 4 bit exponent, and a sign for each. Assume that all numbers are chopped rather than rounded.

1a. ( 5) Give the machine representation and a base 10 representation for machine epsilon, the smallest nonzero positive number which, added to 1, gives a number different from 1.

**Answer:** Since the machine chops, $1.0000 + 0.0001 = 1.0001$, but if anything smaller is added to 1, the answer will be 1.

So machine epsilon is $1/16$ in decimal, which has a machine representation of $+1.0000$ for the mantissa and $-0100$ for the exponent.

1b. ( 5) Which machine number is closest to $\pi$?

**Answer:** $3.14159... = 3 + 1/8 + ...$, which, in binary, is $11.001 = 1.1001 \times 2^1$.Therefore, 3.125 is the closest machine number, and its machine representation would be $+1.1001$ for the mantissa and $+0001$ for the exponent.

2. (5) Suppose I have measured the sides of a rectangle as $3.2 \pm .005$ and $4.5 \pm .005$. Give a bound on the relative error in $A = 3.2 * 4.5$ as an approximation to the area of the rectangle.

**Answer:** The absolute value of the relative error in 3.2 is bounded by $r = .005/3.195$. The absolute value of the relative error in 4.5 is bounded by $s = .005/4.495$. So the absolute value of the relative error in the answer is (approximately) bounded by $r + s = 0.0016 + 0.0011 = .0027$.

3. (5) Define backward error analysis.

**Answer:** It is the process of bounding the distance between the given problem and the problem actually solved.