For this page of the quiz, assume you have a base 2 computer that stores floating point numbers using a 5 bit normalized mantissa (x.xxxx), a 4 bit exponent, and a sign for each. Assume that all numbers are chopped rather than rounded.

1a. (5) Give the machine representation and a base 10 representation for machine epsilon, the smallest nonzero positive number which, added to 1, gives a number different from 1.

**Answer:** Since the machine chops, \(1.0000 + 0.0001 = 1.0001\), but if anything smaller is added to 1, the answer will be 1.

So machine epsilon is \(1/16\) in decimal, which has a machine representation of \(+1.0000\) for the mantissa and \(-0001\) for the exponent.

1b. (5) Which machine number is closest to \(\pi\)?

**Answer:** \(3.14159... = 3 + 1/8 + \ldots\), which, in binary, is \(11.0001 = 1.1001 \times 2^1\). Therefore, \(3.125\) is the closest machine number, and its machine representation would be \(+1.1001\) for the mantissa and \(+0001\) for the exponent.

2. (5) Suppose I have measured the sides of a rectangle as \(3.2 \pm .005\) and \(4.5 \pm .005\). Give a bound on the relative error in \(A = 3.2 \times 4.5\) as an approximation to the area of the rectangle.

**Answer:** The absolute value of the relative error in 3.2 is bounded by \(r = .005/3.195\). The absolute value of the relative error in 4.5 is bounded by \(s = .005/4.495\). So the absolute value of the relative error in the answer is (approximately) bounded by \(r + s = 0.0016 + 0.0011 = .0027\).

3. (5) Define backward error analysis.

**Answer:** It is the process of bounding the distance between the given problem and the problem actually solved.