1. (10) Let

\[ I = \int_2^3 \int_{-1}^x x^2 \cos(xy^2) \, dy \, dx. \]

Given a Matlab integration function `quad('f',a,b, tol)` that computes an approximation to

\[ \int_a^b f(t) \, dt \]

within `tol` of the true value, write code to compute an approximation within 10^{-3} of \( I \).

(Grading: 7 points for an approximation; 10 points for achieving the error tolerance.)

**Answer:** We want to compute a number \( Q \) so that \( |I - Q| < tol \). Now, \( I \) is the integral of the function

\[ f(x) = \int_{-1}^x x^2 \cos(xy^2) \, dy \]

and we don’t compute \( f(x) \) exactly. Instead, we compute an approximation to it, \( \tilde{f}(x) \), where \( |f(x) - \tilde{f}(x)| < \delta \), where \( \delta \) is the tolerance we give to `quad` when forming \( f \). So, if we tell `quad` to integrate \( \tilde{f} \) to a tolerance of \( \epsilon \), then our total error is

\[ |I - Q| \leq \epsilon + \int_2^3 \int_{-1}^x |f(x) - \tilde{f}(x)| \, dy \, dx \leq \epsilon + \delta \int_2^3 \int_{-1}^x \, dy \, dx. \]

Any choice of \( \epsilon \) and \( \delta \) that keep this number less than \( tol \) is fine. For instance, \( \epsilon = \delta = 10^{-4} \), works, but is a little conservative.

\[ I = \text{quad('f',2,3,1.e-4);} \]

```matlab
function a = f(x)
global xx
xx = x
a = quad('g',-1,x,1.e-4);
end
```

```matlab
function a = g(y)
global xx
xx = x^2*cos(x*y^2)
end
```

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2. (10) Write Matlab statements to compute the product of two matrices, A and B, using the outer product formulation of summing columns of A times rows of B.

**Answer:**

```matlab
[m,n]=size(A);
[n,p]=size(B);
C = zeros(m,p);
for i=1:n,
    C = C + A(:,i)*B(i,:);
end
```