AMSC/CMSC 460 Quiz 6 , Fall 2002

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, communication with others, scratchpaper, etc.

Name ____

Student number ________________________________

1. (10) Write Matlab code to efficiently form the matrix-vector product Gx for any vector x of length 8, when

$$G = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & z & z^2 & z^3 & z^4 & z^5 & z^6 & z^7 \\ 1 & z^2 & z^4 & z^6 & 1 & z^2 & z^4 & z^6 \\ 1 & z^3 & z^6 & z^1 & z^4 & z^7 & z^2 & z^5 \\ 1 & z^4 & 1 & z^4 & 1 & z^4 & 1 & z^4 \\ 1 & z^5 & z^2 & z^7 & z^4 & z & z^6 & z^3 \\ 1 & z^6 & z^4 & z^2 & 1 & z^6 & z^4 & z^2 \\ 1 & z^7 & z^6 & z^5 & z^4 & z^3 & z^2 & z^1 \end{bmatrix}$$

and $z = e^{-2\pi i/8}$. (Note that G is the FFT matrix of size 8 with a column reordering.) To receive full credit, your code should require fewer than 21 multiplications.

This was not the problem I intended to give, but here is a correct answer to it:

z	z^2	z^3	z^4	z^5	z^6	z^7	z^8
z	$-i$	z^3	-1	$-z$	i	$-z^3$	1

$$Gx = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & z & -i & z^3 & -1 & -z & i & -z^3 \\ 1 & -i & -1 & i & 1 & -i & -1 & i \\ 1 & z^3 & i & z & -1 & -z^3 & -i & -z \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -z & -i & -z^3 & -1 & z & i & z^3 \\ 1 & i & -1 & -i & 1 & i & -1 & -i \\ 1 & -z^3 & i & -z & -1 & z^3 & -i & z \end{bmatrix} x$$

$$
= \begin{bmatrix} x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 \\ x_1 + z(x_2 - x_6) + z^3(x_4 - x_8) + i(x_7 - x_3) - x_5 \\ x_1 + i(x_8 + x_4 - x_2 - x_6) + x_5 - x_3 - x_7 \\ x_1 - x_5 + i(x_3 - x_7) + z(x_4 - x_8) + z^3(x_2 - x_6) \\ x_1 - x_2 + x_3 - x_4 + x_5 - x_6 + x_7 - x_8 \\ x_1 - x_5 + i(x_7 - x_3) + z(x_6 - x_2) + z^3(x_8 - x_4) \\ x_1 + x_5 - x_3 - x_7 + i(x_2 + x_6 - x_4 - x_8) \\ x_1 - x_5 + i(x_3 - x_7) + z(x_8 - x_4) + z^3(x_6 - x_2) \end{bmatrix}
$$

After saving z^3 in a variable, the Matlab code for this algorithm requires 16 multiplications.

2. (10) Let

$$
A = \begin{bmatrix} 1 & 2 & 1.0 \\ 0 & -1 & 2.5 \\ 5 & 3 & -.5 \end{bmatrix}
$$

Compute the LU decomposition of A using column pivoting. In your answer, include P, L, and U. (You can leave some expressions in form that a calculator can compute, but do enough arithmetic so that you can determine P.)

Answer:

$$
P = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
$$

$$
L = \begin{bmatrix} 1.0000 & 0 & 0 \\ 0.2000 & 1.0000 & 0 \\ 0 & -0.7143 & 1.0000 \end{bmatrix}
$$

$$
U = \begin{bmatrix} 5.0000 & 3.0000 & -0.5000 \\ 0 & 1.4000 & 1.1000 \\ 0 & 0 & 3.2857 \end{bmatrix}
$$