CMSC/AMSC 460 Fall 2007
Homework 7
Due Tuesday, December 11, before class begins
25 points

The Problem we discussed on the first day of class:
(Kahaner, Moler, and Nash, Problem 8-15)

The speed of sound in ocean water depends on

- pressure.
- temperature.
- salinity.

These vary with depth in complicated ways.

Let z be the depth (in feet) under the ocean surface. (The z axis points downward!) Someone has measured these values for $c(z)$, the speed of sound (in ft/sec) at depth z:

c(0) = 5042	c(500) = 4995	c(1000) = 4948
c(1500) = 4887	c(2000) = 4868	c(2500) = 4863
c(3000) = 4865	c(3500) = 4869	c(4000) = 4875
c(5000) = 4875	c(6000) = 4887	c(7000) = 4905
c(8000) = 4918	c(9000) = 4933	c(10000) = 4949
c(11000) = 4973	c(12000) = 4991	

Since the speed of sound varies with depth, sound rays travel in curved paths. (It's like the refraction of light when you look into a fishbowl.)

We make a mathematical model for the paths using Snell's Law. Suppose we broadcast a sound from a point $x = 0, z = z_0$ and let $z(x)$ be the depth of the ray when it is a horizontal distance x from the source.

Let $\theta(x)$ denote the angle between the tangent to $z(x)$ and the horizontal axis:

$$\tan\theta(x) = \frac{dz}{dx}\,.$$

Snell's Law says

$$\frac{\cos\theta}{c(z)} = a$$

where a is a constant.

Putting all of this information together by differentiating these equations, we can obtain the model

$$\begin{aligned}
\frac{d^2z}{dx^2} &= -\frac{c'(z)}{a^2c(z)^3}, \\
a^2 &= \left(\frac{\cos\theta(0)}{c(z_0)}\right)^2, \\
z(0) &= z_0, \\
\frac{dz}{dx}(0) &= \tan\theta_0.
\end{aligned}$$

a) (10) Use `ode45` to trace z for the ray beginning at $z_0 = 2000$ ft and $\theta(0) = 5.4^o$.

- Use a spline to evaluate $c(z)$ and $c'(z)$ wherever they are needed.
- Follow the ray for 24 nautical miles (1 nautical mile = 6076 ft) and plot your solution $z(x)$, $x \in [0, 24mi]$.
- Remember that Matlab trig functions use radians, not degrees.
- Your value for z at $24mi$ should be close to 3000.

b) (5) Now suppose that a sound source at a depth of 2000 ft transmits to a receiver $\hat{x}$ =24 miles away, at a depth of 3000 ft. Write a Matlab function `depth(theta)` that traces the ray with initial angle `theta` and $z_0 = 2000$ ft, and returns the value $z(\hat{x}) - 3000$. Print a table of values of this function for `theta` in the range -10 to 10 degrees.

c) (10) Use `fzero` with starting values obtained from part b to find 4 rays with angles between -10^o and 10^o that pass through the receiver.

What to submit: Hand in

- **a listing of your main program and any Matlab functions you wrote. Each of these should have some documentation: name, date, purpose, description of parameters for any functions, a few words about how it works.**
- **The output your program produces, designed to be easy to read and understand.**
- **the plot for part a, well-labeled by Matlab.**