AMSC/CMSC 460 Quiz 1 , Fall 2007

For this page of the quiz, assume you have a base 2 computer that stores floating point numbers using a 5 bit normalized mantissa (x.xxxx), a 4 bit exponent, and a sign for each. Assume that all numbers are chopped rather than rounded.

1. (10) Consider the the following code fragment:

```
x = 1;
for j=1:2^(20),
  x = x + delta;
end
```

For the computer specified above, what is the largest value of `delta` for which the final value of `x` is 1? Explain your reasoning.

Answer: Since this machine only stores 5 bits, the machine number just greater than 1 is $1.0001_2 = 1 + 1/16$.

In the first iteration of the loop, if `delta = 1/16`, then `x` will change from 1 to 1.0001_2.

If `delta` is any number between 0 and 1/16, then (because of chopping), `x` will not change its value.

So `x` is constant for any positive number less than 1/16.

The largest machine-representable number less than 1/16 is

$$1.1111_2 \times 2^{-5} = 2^{-5}(1 + 1/2 + 1/4 + 1/8 + 1/16) = 1/16 - 1/512.$$

2. Consider the equation $x^2 - .81 = 0$.

(a) (5) What is the relative error in the values $x_1 = .85$, $x_2 = -.85$ as approximations to the two solutions to the equation?

Answer: The two relative errors are equal:

$$\left|\frac{.9 - .85}{.9}\right| = \frac{5}{90}.$$

(b) (5) Give a backward error bound for $x_1 = .91$, $x_2 = -.91$ as approximations to the two solutions to the equation.

Answer: For backward error, we report a bound on the distance between the problem we solved and the problem we wanted to solve. We have solved the problem

$$x^2 - (.91)^2 = 0,$$

when we wanted to solve the problem

$$x^2 - .81 = 0,$$

so we have changed the constant term in the polynomial by

$$.81 - (.91)^2.$$