AMSC/CMSC 460   Quiz 5   ,   Fall 2007

1. In the last in-class exercise, you experimented with taking the discrete Fourier transform (or discrete cosine transform) of the sunspot data and then zeroing out some small components.

1a. (5) What do the components that are large in magnitude tell you about sunspots?

Answer: The sunspot data is dominated by cycles whose period is identified by the large magnitude components of the transform.

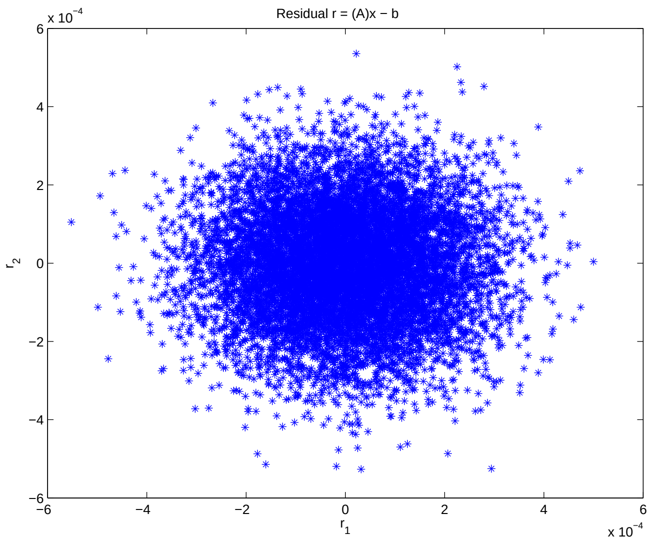
1b. (5) Give two reasons for zeroing out small components of the transform of a series of data like that for sunspots.

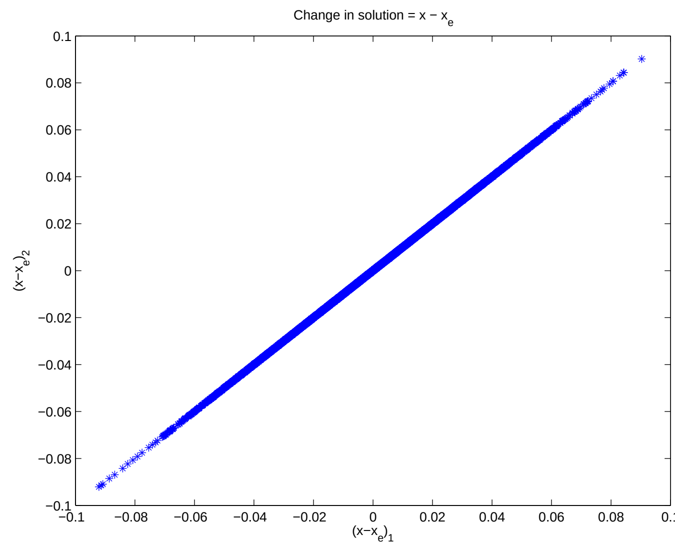
Answer:

- Seeing if the data supports the hypothesis that there are only a few significant cyclical components in the data.
- Compressing the data, by only saving the nonzeros.
- Removing noise by zeroing components that correspond to noise.

2. (10) Consider a linear system $\mathbf{Ax} = \mathbf{b}$ where $\mathbf{A}$ is 2×2. Suppose we perturb the problem, by changing it to $(\mathbf{A} + \mathbf{E})\mathbf{x}_e = \mathbf{b}$, where $\|\mathbf{E}\|_1/\|\mathbf{A}\|_1 = 10^{-4} \equiv \delta$. We repeat this experiment 10,000 times with different values of $\mathbf{E}$ (all with $\|\mathbf{E}\|_1/\|\mathbf{A}\|_1 \approx \delta$) to get the figures above. Assuming that $\|x\|_1 = 1$, use the following fact to estimate the condition number κ of $\mathbf{A}$:

$$\frac{\|\mathbf{x} - \mathbf{x}_e\|}{\|\mathbf{x}\|} \leq \frac{\kappa(\mathbf{A})}{1 - \kappa(\mathbf{A})\delta}\delta.$$





Answer: We see from the graph that the maximum change in the solution is approximately

$$\|\mathbf{x} - \mathbf{x}_e\| = |.1| + |.1| = .2.$$

Therefore, our inequality says that

$$.2 \leq \frac{\kappa(\mathbf{A})}{1 - \kappa(\mathbf{A})10^{-4}} 10^{-4}.$$

We solve this inequality for κ:

$$2000 \leq \frac{\kappa(\mathbf{A})}{1 - \kappa(\mathbf{A})10^{-4}}$$

$$2000 - .2\kappa(\mathbf{A}) \leq \kappa(\mathbf{A})$$

$$2000 \leq 1.2\kappa(\mathbf{A})$$

$$\kappa(\mathbf{A}) \geq \frac{2000}{1.2} = 1667.$$