1. (10) Write Matlab code to use \texttt{fzero} to find an approximate solution to the problem

\[
F(x) = \int_0^x g(t)dt = 5.
\]

You may assume that there is a Matlab function \texttt{g.m} that evaluates \(g(t)\), that \(F(1) < 5\) and that \(F(2) > 5\). Recall that \texttt{fzero} finds a point \(x\) so that \(f(x) = 0\). It takes two arguments: the first defines the function \(f\) and the second is a vector of length 2 where \(f\) evaluated at the first component differs in sign from \(f\) evaluated at the second.

\textbf{Answer:}

\[
x = \texttt{fzero(@F,[1,2])};
\]

\begin{verbatim}
function y = F(x)
y = quad(@g,0,x) - 5;
\end{verbatim}

2. (10) Apply one step of Newton’s method to solve the nonlinear equation

\[
\begin{bmatrix}
x_1^2 + x_2^2 - 1 \\
cos(x_2)
\end{bmatrix} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

using a starting guess of \(x_1 = 1/2, x_2 = \pi/4\). Your answer should be either two numbers (the values of \(x_1\) and \(x_2\) after the iteration), or computable formulas that Matlab could use to get these numbers.

(Recall that \(\cos(\pi/4) = \sin(\pi/4) = \sqrt{2}/2\).)

\textbf{Answer:} The Jacobian matrix is

\[
J(x) = \begin{bmatrix}
2x_1 & 2x_2 \\
0 & -\sin(x_2)
\end{bmatrix}.
\]

Therefore, one step of Newton’s method computes

\[
x \leftarrow x - J(x)\backslash \begin{bmatrix}
x_1^2 + x_2^2 - 1 \\
cos(x_2)
\end{bmatrix},
\]

or

\[
x \leftarrow \begin{bmatrix}
1/2 \\
\pi/4
\end{bmatrix} - \begin{bmatrix}
1 & \pi/2 \\
0 & -\sqrt{2}/2
\end{bmatrix}\backslash \begin{bmatrix}
1/4 + (\pi/2)^2 - 1 \\
\sqrt{2}/2
\end{bmatrix}.
\]