AMSC/CMSC 460 Quiz 7 , Fall 2007

1. (10) Write Matlab code to use `fzero` to find an approximate solution to the problem

$$F(x) = \int_0^x g(t)dt = 5.$$

You may assume that there is a Matlab function `g.m` that evaluates $g(t)$, that $F(1) < 5$ and that $F(2) > 5$. Recall that `fzero` finds a point $x$ so that $f(x) = 0$. It takes two arguments: the first defines the function $f$ and the second is a vector of length 2 where $f$ evaluated at the first component differs in sign from $f$ evaluated at the second.

Answer:

```
x = fzero(@F,[1,2]);

function y = F(x)
y = quad(@g,0,x) - 5;
```

2. (10) Apply one step of Newton's method to solve the nonlinear equation

$$\begin{bmatrix} x_1^2 + x_2^2 - 1 \\ \cos(x_2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

using a starting guess of $x_1 = 1/2$, $x_2 = \pi/4$. Your answer should be either two numbers (the values of $x_1$ and $x_2$ after the iteration), or computable formulas that Matlab could use to get these numbers.
(Recall that $\cos(\pi/4) = \sin(\pi/4) = \sqrt{2}/2$.)

Answer: The Jacobian matrix is

$$\mathbf{J}(\mathbf{x}) = \begin{bmatrix} 2x_1 & 2x_2 \\ 0 & -\sin(x_2) \end{bmatrix}.$$

Therefore, one step of Newton's method computes

$$\mathbf{x} \leftarrow \mathbf{x} - \mathbf{J}(\mathbf{x}) \backslash \begin{bmatrix} x_1^2 + x_2^2 - 1 \\ \cos(x_2) \end{bmatrix},$$

or

$$\mathbf{x} \leftarrow \begin{bmatrix} 1/2 \\ \pi/4 \end{bmatrix} - \begin{bmatrix} 1 & \pi/2 \\ 0 & -\sqrt{2}/2 \end{bmatrix} \backslash \begin{bmatrix} 1/4 + (\pi/2)^2 - 1 \\ \sqrt{2}/2 \end{bmatrix}.$$