AMSC/CMSC 660 Quiz 1 , Fall 2003

For Problem 1, assume you have a base 10 computer that stores floating point numbers using a 5 digit normalized mantissa (x.xxxx), a 4 digit exponent, and a sign for each. Assume that all numbers are chopped rather than rounded.

1a. (5) For this machine, what is machine epsilon, the smallest nonzero positive number which, added to 1, gives a number different from 1?

Answer: $1.0000 + 0.0001 = 1.0001$, but $1.0000 + 0.0000999...$ would be chopped to 1.0000, so machine epsilon is 10^{-4}.

1b. (5) What is the smallest positive number that can be represented exactly in this machine?

Answer: The smallest positive normalized mantissa is 1.0000, and the smallest exponent is -9999, so the number is 1×10^{-9999}. (Note that this is much smaller than machine epsilon.)

2. Suppose I have measured the sides of a rectangle as $3.2 \pm .005$ and $4.5 \pm .005$, and I compute an approximation to the area as $A = 14$.
a. (5) Give a forward error bound for the computation.

Answer: Ordinarily, relative error bounds add when we do multiplication, but the dominant error in this computation was the rounding of the answer from $3.2 \times 4.5 = 14.4$ to 14 (Perhaps we could store only 2 decimal digits). Therefore, one way to express the forward error bound is that the true answer lies between 13.5 and 14.5.

b. (5) Give a backward error bound.

Answer: Again, there are many correct answers. For example, we have exactly solved the problem $3.2 \times (14/3.2)$, or 3.2×4.37, so we have changed the second piece of data by 0.13.