

1. (10) Find an orthogonal matrix Q and a number z so that

$$Q \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} z \\ 0 \end{bmatrix}.$$

Answer: We will compute a Givens matrix by setting

$$c = \frac{3}{\sqrt{9+16}}, \quad s = \frac{4}{\sqrt{25}}.$$

Then if

$$Q = \begin{bmatrix} 3/5 & 4/5 \\ 4/5 & -3/5 \end{bmatrix},$$

then

$$Q \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$$

so $z = 5$, the norm of the original vector.

2. (10) Write a column oriented algorithm to solve the linear system $Ax = b$ where A is an $n \times n$ nonsingular lower triangular matrix. (5 points for a correct algorithm that accesses A sequentially by rows instead of columns.)

Answer:

```
x = b;
for i=1:n,
    x(i) = x(i) / a(i,i);
    x(i+1:n) = x(i+1:n) - a(i+1:n,i)*x(i);
end
```