

1. (10) Suppose we have factored the $n \times n$ matrix A as $PA = LU$ and now we want to solve the linear system formed by adding one row and one column to A to make a matrix

$$A_{new} = \begin{bmatrix} a_{1,1} & \cdots & a_{1,n} & a_{1,n+1} \\ a_{2,1} & \cdots & a_{2,n} & a_{2,n+1} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n,1} & \cdots & a_{n,n} & a_{n,n+1} \\ a_{n+1,1} & \cdots & a_{n+1,n} & a_{n+1,n+1} \end{bmatrix}.$$

Express A_{new} as

$$A_{new} = \begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix} - ZV^T$$

(where Z and V are rank-2 matrices) so that the Sherman-Morrison-Woodbury Formula could be applied.

2. (10) Write a Matlab program to apply 5 iterations of Newton's method to the problem

$$\min_x (x_1 - 5)^4 + (x_2 + 1)^4 - x_1 x_2$$

with a steplength of 1 (i.e, step in the Newton direction without a linesearch) and with an initial starting guess of $x = [1, 2]^T$.