AMSC/CMSC 660 Quiz 4 , Fall 2003

1. (10) Suppose we have factored the $n \times n$ matrix A as $PA = LU$ and now we want to solve the linear system formed by adding one row and one column to A to make a matrix

$$A_{new} = \begin{bmatrix} a_{1,1} & \dots & a_{1,n} & a_{1,n+1} \\ a_{2,1} & \dots & a_{2,n} & a_{2,n+1} \\ \vdots & \vdots & \vdots & \vdots \\ a_{n,1} & \dots & a_{n,n} & a_{n,n+1} \\ a_{n+1,1} & \dots & a_{n+1,n} & a_{n+1,n+1} \end{bmatrix}.$$

Express A_{new} as

$$A_{new} = \begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix} - ZV^T$$

(where Z and V are rank-2 matrices) so that the Sherman-Morrison-Woodbury Formula could be applied.

Answer:

$$\begin{aligned} A_{new} &= \begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} a_{n+1,1} & \dots & a_{n+1,n} & a_{n+1,n+1} - 1 \end{bmatrix} + \begin{bmatrix} a_{1,n+1} \\ a_{2,n+1} \\ \vdots \\ a_{n,n+1} \\ 0 \end{bmatrix} \begin{bmatrix} 0 & \dots & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} A & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & a_{1,n+1} \\ 0 & a_{2,n+1} \\ \vdots & \vdots \\ 0 & a_{n,n+1} \\ 1 & a_{n+1,n+1} - 1 \end{bmatrix} \begin{bmatrix} a_{n+1,1} & \dots & a_{n+1,n} & 0 \\ 0 & \dots & 0 & 1 \end{bmatrix} \end{aligned}$$

So we can take

$$Z = -\begin{bmatrix} 0 & a_{1,n+1} \\ 0 & a_{2,n+1} \\ \vdots & \vdots \\ 0 & a_{n,n+1} \\ 1 & a_{n+1,n+1} - 1 \end{bmatrix}; V^T = \begin{bmatrix} a_{n+1,1} & \dots & a_{n+1,n} & 0 \\ 0 & \dots & 0 & 1 \end{bmatrix}$$

2. (10) Write a Matlab program to apply 5 iterations of Newton's method to the problem

$$\min_x (x_1 - 5)^4 + (x_2 + 1)^4 - x_1 x_2$$

with a steplength of 1 (i.e, step in the Newton direction without a linesearch) and with an initial starting guess of $x = [1, 2]^T$.

Answer:

```
x = [1;2];
for i=1:5,
   g = [4*(x(1) - 5)^3 - x(2); 4*(x(2) + 1)^3 - x(1)];
   H = [12*(x(1) - 5)^2, -1; -1,  12*(x(2) + 1)^2];
   p = -H \ g;
   x = x + p;
end
```