

1a. (5) Suppose we have a QR decomposition of a matrix A of dimension $m \times n$ and then we add a row to A . How many Givens rotations does it take to form the QR decomposition of the new matrix?

Answer: n Givens rotations. (See notes p.21)

1b (5) Suppose we have an SVD of a nonsingular matrix A of dimension $n \times n$: $A = U\Sigma V^T$. Given the SVD, how many multiplications would it take to solve the linear system $Ax = b$? (In your answer, make clear how you would use the factors to solve the problem.)

Answer:

Form $y = U^T b$ n^2 multiplications
 Form $z = \Sigma^{-1}y$ n multiplications
 Form $x = Vz$ n^2 multiplications
 Total: $2n^2 + n$ multiplications

2. (10) Write code to estimate the area of the circle centered at $(0,0)$ of radius 1 by Monte-Carlo integration using 1000 points.

Answer:

```
% Estimate area of quarter circle and multiply by 4.
n = 1000;
z = rand(n,2);
zz = z(:,1).^2 + z(:,2).^2;
n1 = sum(zz <= 1);
area = (n1/n)*4;
```

Alternatively,

```
n = 1000;
z = rand(n,1);
zz = sqrt(1-z.^2)    % Integrate this function from 0 to 1
s = sum(zz)/n;
area = 4*s;
```