AMSC/CMSC 660 Quiz 6 , Fall 2003

1. (10) Recall the Matlab demonstration program `travel.m` for computing an approximate solution to a traveling salesperson problem using a randomized algorithm. What changes would you need to make to that program to change it into a Metropolis algorithm?

Answer:

1. Add an outer loop that decreases a temperature parameter.
2. When an exchange of two cities is generated, accept the exchange if it decreases the total length of the circuit (as is done currently) but if it increases it, accept it with some probability that increases with temperature.

(This is somewhat oversimplified, but enough for the quiz.)

2. (10) Suppose we have used the Adams-Bashforth and Adams-Moulton methods of order 3 to form two estimates of $y(t_{n-1})$, the solution to a differential equation. These formulas are:

$$\begin{aligned} y_{n+1}^{ab} &= y_n + \frac{h}{12}(23f_n - 16f_{n-1} + 5f_{n-2}) \; error: \frac{3h^4}{8}y^{(4)}(\eta) \\ y_{n+1}^{am} &= y_n + \frac{h}{12}(5f_{n+1} + 8f_n - f_{n-1}) \; error: -\frac{h^4}{24}y^{(4)}(\eta) \end{aligned}$$

How would you estimate the local error in the Adams-Moulton formula? How would you use that estimate to change h in order to keep the estimated local error less than a user-supplied local error tolerance τ without taking steps smaller than necessary?

Answer: Subtracting, we obtain

$$y_{n+1}^{ab} - y_{n+1}^{am} = \frac{3h^4}{8}y^{(4)}(\eta) - (-\frac{h^4}{24}y^{(4)}(\nu))$$

where η, ν are in the interval containing y_{n+1}^{ab}, y_{n+1}^{am}, and the true value. Since $3/8 + 1/24 = 10/24$, the error in AM can be estimated as $\epsilon = |y_{n+1}^{ab} - y_{n+1}^{am}|/10$. Now, if $\epsilon > \tau$, we might reduce h by a factor of 2 and retake the step. If $\epsilon << \tau$, we might double h in preparation for the next step (expecting that the local error might increase by a factor of 2^4).