AMSC/CMSC 660 Quiz 7 , Fall 2003

1. (10) Suppose we solve the problem

$$\begin{aligned} y'' &= 6y' - ty + y^2 \\ y(0) &= 5 \\ y(1) &= 0 \end{aligned}$$

using the finite difference method, approximating $y_i \approx y(ih)$ where $h = .01$, $i = 0, \ldots, 100$. We will use a nonlinear equation solver on the system $F(y) = 0$, where there are 99 unknowns and 99 equations. Write the equations for $F(y)$.

Answer: For $i = 1, \ldots, 99$,

$$F_i(y) = \frac{y_{i-1} - 2y_i + y_{i+1}}{h^2} - 6\frac{y_{i+1} - y_{i-1}}{2h} + ihy_i - y_i^2 ,$$

where $y_0 = 5$ and $y_{100} = 0$.

2. (10) Apply one step of Newton's method (with step-length equal to 1) to the problem

$$\min_x x_1^4 + x_2(x_2 - 1)$$

starting at the point $x_1 = 2$, $x_2 = -1$.

Answer:

$$f(x) = x_1^4 + x_2(x_2 - 1)$$

$$g(x) = \begin{bmatrix} 4x_1^3 \\ 2x_2 - 1 \end{bmatrix}, \quad H(x) = \begin{bmatrix} 12x_1^2 & 0 \\ 0 & 2 \end{bmatrix}.$$

Step 1:

$$p = -\begin{bmatrix} 12x_1^2 & 0 \\ 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 4x_1^3 \\ 2x_2 - 1 \end{bmatrix} = -\begin{bmatrix} 48 & 0 \\ 0 & 2 \end{bmatrix}^{-1} \begin{bmatrix} 32 \\ -3 \end{bmatrix} = \begin{bmatrix} -32/48 \\ +3/2 \end{bmatrix}$$

so

$$x \leftarrow \begin{bmatrix} 2 \\ -1 \end{bmatrix} + \begin{bmatrix} -2/3 \\ +3/2 \end{bmatrix} = \begin{bmatrix} 4/3 \\ 1/2 \end{bmatrix}$$