AMSC/CMSC 660 Quiz 1 , Fall 2004

For Question 1, assume you have a base 2 computer that stores floating point numbers using a 6 digit normalized mantissa (x.xxxxx), a 4 digit exponent, and a sign for each. Assume that all numbers are chopped rather than rounded.

1a. (5) For this machine, what is machine epsilon, the smallest nonzero positive number which, added to 1, gives a number different from 1?

Answer: $1.00000 + .00001 = 1.00001$, but adding anything less than .00001 to 1 produces an answer of 1. Therefore, for this machine, machine epsilon is $.00001_2 = 2^{-5}$.

1b. (5) What mantissa and exponent are stored for the value 1/10? Hint:

$$\frac{1}{10} = \frac{1}{16} + \frac{1}{32} + \frac{1}{256} + \frac{1}{512} + \frac{1}{4096} + \frac{1}{8192} + ...?$$

Answer: $1/10 = 1.1001100..._2 \times 2^{-4}$, so the mantissa is $+1.10011$ and the exponent is $-4 = -0100_2$.

2. (5) Suppose that you have measured the length of the side of a cube as $(3.00 \pm .005)$ meters. Give an estimate of the volume of the cube and a (good) bound on the absolute error in your estimate.

Answer: The estimated volume is $3^3 = 27m^3$.
The relative error in a side is bounded by $z = .005/2.995$.
Therefore, the relative error in the volume is bounded by $3z$ (if we ignore the high order terms), so the absolute error is bounded by $27 * 3z \approx 27 * .005 = .135$.

3. (5) Consider the following Matlab code:

```
x = .1;
sum = 0;
for i=1:100
  sum = sum + x;
end
```

Is the final value of `sum` equal to 10? If not, why not?

Answer: No.

- .1 is not represented exactly, so error occurs in each use of it.
- If we repeatedly add the machine value of .1, the exact machine value for the answer does not always fit in the same number of bits, so additional error is made in storing the answer. (Note that this error would occur even if .1 were represented exactly.)

(Recall that this was the issue in the Patriot Missile disaster.)