

AMSC/CMSC 660 Quiz 2 , Fall 2004

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, other electronic devices, communication with others, scratchpaper, etc.

Name _____

Student number _____

1. Consider the following algorithm for generating random numbers:

Take 5 papers and number them 0 to 4. Put them in a box, and draw one at random. Record the resulting number, and put the paper back in the box.

1a. (5) What is the probability density function for the distribution of samples drawn using this algorithm?

1b. (5) What are the mean and variance of the distribution?

2. (10) Suppose we estimate the integral

$$I = \int_0^1 x^3 dx = \int_0^1 \left(\frac{x^2}{2}\right) (2x) dx$$

using Monte-Carlo integration (n samples) with importance sampling. (A silly example, but I hope it contributes to your understanding.)

2a. What will the variance in the estimates be if we let $p_1(x) = 1$ and choose the sample points for $f_1(x) = x^3$ according to the distribution $p_1(x)$?

2b. What will the variance in the estimates be if we let $p_2(x) = 2x$ and choose the sample points for $f_2(x) = x^2/2$ according to the distribution $p_2(x)$?