AMSC/CMSC 660 Quiz 3 , Fall 2004

1a. (5) Recall the Kenyon, Randall, and Sinclair (KRS) algorithm for counting the number of arrangements of k dimers on a given lattice. At each iteration, either nothing happens or a dimer is added, deleted, or moved. If it has been long enough since our last recording, then the resulting configuration is added to the record. Why don't we record every configuration?

Answer: In order to get good estimates of the counts, the samples that we record must be (almost) uncorrelated. In other words, the probability of getting a particular sample must not depend on the value of the preceding sample.
Note: It is not correct to say that we skip in order to get *random* samples. Some random samples are correlated and some are uncorrelated! For example, suppose we construct a string of numbers by starting with 0. To obtain the next entry in the string, we do this:

- If the latest entry is 0, we add a 0 with probability .8 and a 1 with probability .2.
- If the latest entry is 1, we add a 0 with probability .3 and a 1 with probability .7.

The entries in the string are a *random*, because we don't make the same choices each time we generate a string. But they are *correlated*: for example, the probability of seeing the pattern 00 is much more likely than seeing the pattern 01.

1b. (5) Describe what happens in 1 iteration of a Metropolis algorithm for the Traveling Salesperson Problem (TSP) after a pair of cities is chosen at random.

Answer: Try interchanging the two cities. If the length of the resulting circuit is less than the old, then we keep the new circuit and discard the old. If the length is not less, then we keep the new circuit with a probability that

- increases with temperature,
- decreases with the change in length.

We also might decide to decrease the temperature.

2a. (2) If y is an $n \times 1$ vector and Q is an $n \times n$ real orthogonal matrix, prove that $||Qy||_2 = ||y||_2$.

Answer: Since $Q^TQ = I$, we see that $||Qy||_2^2 = (Qy)^T(Qy) = y^TQ^TQy = y^Ty = ||y||_2^2$, Since norms are nonnegative quantities, take the square root and conclude that $||Qy||_2 = ||y||_2$.

2b. (8) Let A be an $m \times n$ matrix. Write a column-oriented Matlab algorithm for computing

$$s_i = \sum_{j=1}^{n} |a_{ij}|$$

for $i = 1, 2, \ldots, m$.

Answer:

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s = zeros(m,1);
for j=1:n,
   s = s + abs(A(:,j));
end
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