AMSC/CMSC 660 Quiz 4 , Fall 2004

1. (10) Fill in each box with the name of a matrix decomposition that can be used to efficiently solve the given problem in a stable manner.

Find the null space of a matrix	QR (fast; relatively stable) SVD (slower but more reliable)
Solve a least squares problem when the matrix is well conditioned	QR[1]
Determine the rank of a matrix	RR-QR (fast, relatively stable) SVD (slower but more reliable)
Find the determinant of a matrix	LU with pivoting
Determine whether a symmetric matrix is positive definite	Cholesky[2] Eigendecomposition (slower but more reliable)

1. Don't try QR if the matrix is *not* well-conditioned. Use the SVD methods that we talked about.
2. The LL^T version of Cholesky will break down if the matrix has a negative eigenvalue by taking the square root of a negative number, so it is a good diagnostic. If the matrix is singular, (positive semi-definite), then you will get a 0 on the main diagonal, but with round-off error, this will be impossible to detect.

2. (10) Let `W = givens(y)` be a Matlab program that takes a 2×1 vector y as input and returns a Givens matrix W that makes Wy have a zero in its 2nd position.

Write a Matlab program that uses `givens` to reduce a matrix of the form

$$A = \begin{bmatrix} x & x & x & x \\ x & x & x & x \\ 0 & x & x & x \\ 0 & 0 & x & x \end{bmatrix}$$

to upper triangular form, where x indicates a nonzero element. (In other words, do a QR decomposition of this matrix, but don't worry about saving Q.)

```
for i=1:3,

   W = givens(A(i:i+1,i));

   % Note that the next instruction just operates on the part
   % of A that changes.  It is wasteful to do multiplications
   % on the rest.

   A(i:i+1,i:n) = W * A(i:i+1,i:n);

end
```