AMSC/CMSC 660 Quiz 7 , Fall 2004

1. (10) Suppose that you have developed an ode model that predicts the amount of profit that you will receive on December 11 if you invest \$1000 today in various components of your business, and that profit depends on 5 parameters $x_1, \ldots, x_5$, so that

$$\begin{aligned} y'(t) &= f(t, y, x), \\ y(0) &= 0, \\ y(1) &= \text{profit on December 11, using parameters } x. \end{aligned}$$

You want to choose those 5 parameters in $x$ to maximize $y(1)$ (which is a scalar value). (Then you will take the money and run.)

What numerical algorithms would you use to solve your problem and how would they pass information to each other? Why did you choose these particular algorithms?

**Answer:** (Note: This problem is like the unquiz on p.7 of the 460 notes.) I would use pattern search to minimize $F(x) = -y(1)$ as a function of $x$. When a function value $F(x)$ is needed, I would call one of Matlab's stiff ode solvers, since I don't know whether the problem is stiff or not, and return the value computed as $y(1)$. The value of $x$ would need to be passed to the function that evaluates $f$ for the ode solver; one way to do this is to use a global variable.

I chose pattern search because it has proven convergence and does not require derivatives of $F$ with respect to $x$. Note that these derivatives are not available for this problem: we can compute derivatives of $y$ with respect to $t$ but not with respect to $x$. And since our value of $y(1)$ is only an approximation, the use of finite differences to estimate derivatives with respect to $x$ would yield values too noisy to be useful.

2. (10) Let

$$\begin{aligned} y' &= y^2 - 5t \\ y(0) &= 1 \end{aligned}$$

Apply a PECE scheme to this problem, using Euler and Backward Euler with a stepsize $h = .1$, to obtain an approximation for $y(.2)$.

**Answer:** Recall that Euler's method is

$$y_{n+1} = y_n + hf(t_n, y_n)$$

and Backward Euler is

$$y_{n+1} = y_n + hf(t_{n+1}, y_{n+1})$$

In the first step, we obtain an approximation to $y(.1)$:

$$\begin{aligned} P: &\quad y_{1p} = 1 + .1(1^2) = 1.1 \\ E: &\quad f_{1p} = (1.1)^2 - .5 = .71 \\ C: &\quad y_{1c} = 1 + .1 * .71 = 1.071 \\ E: &\quad f_{1c} = (1.071)^2 - .5 = .647041 \end{aligned}$$

In the second step, we obtain an approximation to $y(.2)$:

$$\begin{aligned} P: &\quad y_{2p} = y_{1c} + .1f_{1c} = 1.071 + .1(.647041) = 1.1357041 \\ E: &\quad f_{2p} = f(.2, y_{2p}) = y_{2p}^2 - 5 * .2 = .289823803 \\ C: &\quad y_{2c} = y_{1c} + .1f_{2p} = 1.09998238 \\ E: &\quad f_{2c} = f(.2, y_{2c}) = .20996124 \end{aligned}$$