AMSC/CMSC 660 Quiz 4 , Fall 2006

1. (10) Write Matlab code using `rand` to generate a random number from the following distribution:

> The probability that the number is 0 is 0.6.
> The probability that the number is 1 is 0.4.

(In other words, if $p(x)$ is the probability density function, then $p(0) = 0.6$ and $p(1) = 0.4$.)

Answer:

```
% In this code,
%   z is a sample from a uniform distribution on [0,1].
%   y is a sample from the desired distribution.

z = rand(1);
if (z < .6)
   y = 0;
else
   y = 1;
end
```

2. (10) Write MATLAB code to compute the volume of the unit sphere $x_1^2 + x_2^2 + x_3^2 \leq 1$ using Monte Carlo integration. (You may use any of our three methods, although I suggest not using importance sampling because it is harder to write down.)

Answer: Here are implementations of "Method 1" (5 lines of MATLAB code) and "Method 2" (8 lines of code).

```
% "Method 1":  We compute the volume of the unit sphere
% as 8 times the volume of the piece of it in the 1st orthant
% (x_1 >= 0, x_2 >= 0, x_3 >= 0).  Call this piece Omega.
%
% We can estimate the volume of Omega by uniformly sampling points
% in the unit box and counting the proportion of them that are
% also in Omega.
%
% Let N be the number of sample points.

N = 1000 % as an example

% Construct N samples, 3 coordinates each.

x = rand(N,3);

% Compute x_1^2 + x_2^2 + x_3^2 - 1 for each of them.
% (This uses the vector capabilities of Matlab and is faster than
% doing it component by component.)

y = x(:,1).^2 + x(:,2).^2 + x(:,3).^2 - 1;

% Count the number of points in the region, and divide by N
% to get the estimate of the volume of Omega.

Omega_vol = sum(y < 0) / N;

% Multiply by 8 to estimate the volume of the sphere.

Sphere_vol = 8 * Omega_vol;
```

Or:

```
% "Method 2":  We can compute the volume of the unit sphere
% as 2 times the integral from y=-1 to y=1 of
% the integral from x=-sqrt(1-y^2) to x= sqrt(1-y^2)
% of sqrt(1-x^2-y^2).  (See pointer on p. 182)
%
% In other words, we need the integral of sqrt(1-x^2-y^2)
% over the domain that is the unit circle.
%
% Let N be the desired number of sample points.

N = 1000 % as an example

% Construct (approx.) N samples, uniformly distributed in the unit
% quarter circle.
% (Since the volume of the unit box is 1 and the volume of the quarter
% circle is pi/4, we need to sample about 4N/pi points in the box
% to get about N in the quarter circle.)

x = rand(ceil(4*N/pi),2);  % These are uniformly distributed in the square.
test = x(:,1).^2 + x(:,2).^2 -1;
[good_indices,z] = find(test < 0);
x = x(good_indices,:);  % These are uniformly distributed in the quarter circle.

% Find the average function value.

z = sqrt(1-x(:,1).^2 -x(:,2).^2);
averagez = sum(z)/length(z);

% The estimate of the integral is 8 times averagez times the area of the
% quarter circle.

Sphere_vol = 2 * averagez * pi;
```