1a. (5) Verify that the BFGS matrix

\[ B^{(k+1)} = B^{(k)} - \frac{B^{(k)}s^{(k)}s^{(k)T}B^{(k)}}{s^{(k)TB^{(k)}s^{(k)}}} + \frac{y^{(k)}y^{(k)T}}{y^{(k)Ts^{(k)}}}. \]

satisfies the secant condition: \( B^{(k+1)}s^{(k)} = y^{(k)}. \)

1b. (5) Define \( s^{(k)} \) and \( y^{(k)} \). Why are quasi-Newton matrices designed to satisfy the secant condition?

**Answer:**

1a.

\[
B^{(k+1)}s^{(k)} = B^{(k)}s^{(k)} - \frac{B^{(k)}s^{(k)}s^{(k)T}B^{(k)}}{s^{(k)TB^{(k)}s^{(k)}}}s^{(k)} + \frac{y^{(k)}y^{(k)T}}{y^{(k)Ts^{(k)}}}s^{(k)} = B^{(k)}s^{(k)} - \frac{B^{(k)}s^{(k)}s^{(k)T}B^{(k)}}{s^{(k)TB^{(k)}s^{(k)}}}s^{(k)} + \frac{y^{(k)}y^{(k)T}}{y^{(k)Ts^{(k)}}}s^{(k)} = B^{(k)}s^{(k)} - B^{(k)}s^{(k)} + y^{(k)} = y^{(k)}.
\]

1b. For quadratic functions, we have \( Hs^{(k)} = y^{(k)} \), where \( s^{(k)} = x^{(k+1)} - x^{(k)} \) (the change in \( x \)) and \( y^{(k)} = g^{(k+1)} - g^{(k)} \) (the change in gradient). Therefore, we demand the same property from the Quasi-Newton matrix, since it forms a quadratic model for our function.
2. (10) Write a MATLAB program to apply 5 iterations of Newton’s method to the problem

\[
\min_x (x_1 - 2)^4 + (x_2 + 1)^4 - x_1^2 x_2
\]

with a steplength of 1 (i.e., step in the Newton direction without a linesearch) and with an initial starting guess of \( x = [1, 2]^T \).

**Answer:**

```matlab
x = [1,2]';
for k=1:5,
  g = [4*(x(1)-2)^3 - 2*x(1)*x(2);
       4*(x(2)+1)^3 - x(1)^2];
  H = [12*(x(1)-2)^2 - 2*x(2), -2*x(1);
       -2*x(1), 12*(x(2)+1)^2];
  p = -H\g;
  x = x + p;
end
```