AMSC/CMSC 660 Quiz 8 , Fall 2006

1. (10) Let $i = \sqrt{-1}$, and suppose we have a system of differential equations $\mathbf{y}' = \mathbf{y}(t, \mathbf{y})$ with 3 components. Suppose the system has a Jacobian matrix $\mathbf{J}(t, \mathbf{y})$ with eigenvalues

$$\begin{aligned} 4 - t^2, \\ -t - it, \\ -t + it. \end{aligned}$$

For what values of t is the equation stable?

Answer: We need the real parts of all eigenvalues to be negative. This means $4 - t^2 < 0$ and $-t < 0$, so the equation is stable when $t > 2$.

2. Let

$$\begin{aligned} y' &= 10y^2 - 20\,, \\ y(0) &= 1\,. \end{aligned}$$

Apply a PECE scheme to this problem, using Euler and Backward Euler with a stepsize $h = .1$, to obtain an approximation for $y(.1)$.

Answer: $f(t, y) = 10y^2 - 20$.
P: $y^P = y(0) + .1f(0, y(0)) = 1 + .1(-10) = 0$.
E: $f^P = f(.1, y^P) = 10 * 0 - 20 = -20$.
C: $y^C = y(0) + .1f^P = 1 - 2 = -1$.
E: $f^C = f(.1, y^C) = 10 - 20 = -10$.

Note that the predicted and corrected values are quite different, so neither can be trusted; we should reduce the stepsize and recompute. The true value is $y(.1) \approx -0.69$.