

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, other electronic devices, communication with others, scratchpaper, etc.

Name \_\_\_\_\_

1. Suppose we use IEEE standard double precision floating point arithmetic, expressing nonzero numbers as  $\pm z \times 2^p$ , where  $1 \leq z < 2$ ,  $z$  has  $d = 53$  bits of precision, and  $-1022 \leq p \leq 1023$ . Answer the following two questions regarding this representation scheme.

1a. (5 points) Let  $x = 2^k$ . For what integer values of  $k$  can  $x$  be represented exactly?

1b. (5 points) What is the smallest nonzero positive number?

2. (10 points) Consider the math identity

$$1 + \frac{1}{2} + \frac{1}{4} + \cdots + \frac{1}{2^k} = 2 - \frac{1}{2^k}.$$

Consider the following MATLAB code, run in IEEE standard double precision arithmetic.

```
mysum = 1;  
k=1;  
while (mysum < 2)  
    mysum = mysum + 1/2^k;  
    k = k + 1;  
end
```

Despite the identity above, this code terminates. What is the final value of  $k$ ? Explain why termination occurs and, in particular, why it occurs for this particular value of  $k$ .