AMSC/CMSC 660 Quiz 4 , Fall 2009

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, other electronic devices, communication with others, scratchpaper, etc.

Name ____

1. (10) Define a Hamiltonian

$$H(x,p) = .5kx^2 + p^2/(2m),$$

where k and m are given positive constants and x and p are functions of t. Derive the corresponding Hamiltonian system $\boldsymbol{y}' = \boldsymbol{D} \bigtriangledown_y \boldsymbol{H}(\boldsymbol{y}(t)) = \boldsymbol{f}(t,\boldsymbol{y})$, where

$$\boldsymbol{D} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

(In other words, tell me what $\boldsymbol{f}$ is.)

2. (10) MATLAB's ODE solvers include:

- `ode23`: Runge-Kutta, order 2 and 3.
- `ode45`: Runge-Kutta, order 4 and 5.
- `ode113`: Adams-Bashforth-Moulton PECE, order 1 through 12.
- `ode15s`: Gear's method, order 1 through 5.

Choose the most appropriate solver for each of these problems $\boldsymbol{y}' = \boldsymbol{f}(t, \boldsymbol{y})$, with $\boldsymbol{y}(0)$ given. Justify each choice with a sentence.

(a) Need a rough approximation to the solution, the problem is not stiff, and $\boldsymbol{f}$ is inexpensive.

(b) Need a high-accuracy approximation to the solution, the problem is not stiff, and $\boldsymbol{f}$ is expensive (it takes 1 second).

(c) Need a low accuracy approximation to the solution to a very stiff problem and $\boldsymbol{f}$ is expensive.