

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no books, calculators, cellphones, other electronic devices, communication with others, scratchpaper, etc.

Name \_\_\_\_\_

1. (10) Write a sequence of MATLAB commands that explain how to use the QR factorization of the  $m \times n$  matrix  $\mathbf{A}$  to solve the least squares problem

$$\min_{\mathbf{x}} \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2$$

when the rank of  $\mathbf{A}$  is  $n$ . (“Solve” means “compute the optimal  $\mathbf{x}$  and the value of  $\|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2$ .”) Your first command should be `[Q,R] = qr(A)`;

2(a) (7) Let  $\mathbf{B}$  be an  $m \times n$  matrix of rank  $n$  (i.e., it has  $n$  linearly independent columns). Use the QR decomposition of  $\mathbf{B}$  ( $\mathbf{B} = \mathbf{Q}\mathbf{R}$ , where  $\mathbf{Q}$  is  $m \times m$ ) to write an expression (that does not involve  $\mathbf{B}$  or  $\mathbf{B}^T$ ) for the set of all vectors  $\mathbf{x}$  that satisfy the constraint that  $\mathbf{B}^T \mathbf{x} = \mathbf{0}$ .

2(b) (3) Consider the following problem:

$$\min_{\mathbf{x}} \|\mathbf{b} - \mathbf{A}\mathbf{x}\|_2^2$$

subject to the constraint that

$$\mathbf{B}^T \mathbf{x} = \mathbf{0}.$$

Write MATLAB code to solve this problem.