1. (10) Let $\Omega = [0, 1]$ and let 

$$u(x) = e^{5x} + x^2.$$ 

Evaluate $\|u\|_2$, $\|u\|_C$, and $\|u\|_1$. 
2. (10) Consider the differential equation

\[-u'' + 8.125\pi \cot((1 + x)\pi/8)u' + \pi^2 u = -3\pi^2 \text{ on } \Omega = (0, 1)\]

with boundary conditions \(u(0) = -2.0761\), \(u(1) = -2.2929\). Without using a Green’s function or an explicit solution to the problem, tell me about the solution: Does it exist? Is it unique? What are upper and lower bounds on the solution? Justify each of your answers by citing a theorem and verifying its hypotheses. (Hint: One bound can be obtained by comparing the solution to \(u(x) = -3\).)