AMSC/CMSC 661 Quiz 7 , Spring 2005

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. During the quiz you may use your textbook, my notes, and your own notes. No communication with others and no calculators or other electronic devices are permitted.

Name ____

1. (10) We want to solve the ordinary differential equation

$$u' + au = 0$$

where $u : \mathcal{R} \to \mathcal{R}$, $a > 0$ is a scalar, and $u(0)$ is given. Consider the numerical method

$$\frac{u^{n+1} - u^n}{k} + a\frac{u^{n+1} + u^n}{2} = 0$$

where $u^n \approx u(nk)$ and $k > 0$ is the time step. For what range of k values is the numerical method stable?

2. (10) Consider the parabolic initial boundary value problem

$$u_t - u_{xx} = \cos(tx)$$

where $u : [0,1] \times [0,\infty) \to \mathcal{R}$, $u(0,t) = u(1,t) = 0$, and $u(x,0)$ is given. Suppose we use piecewise linear finite elements in the x variable to discretize, with nodes at $x_j = jh$, $j = 0, \ldots, M$. The jth basis function is ϕ_j, where

$$\phi_j(x) = \begin{cases} \frac{x-x_{j-1}}{x_j-x_{j-1}} & x \in [x_{j-1}, x_j] \\ \frac{x-x_{j+1}}{x_j-x_{j+1}} & x \in [x_j, x_{j+1}] \\ 0 & \text{otherwise} \end{cases}$$

Let $u_j(t) \approx u(jh,t)$ and form a system of equations for $u(t) = [u_1(t), \ldots, u_{M-1}(t)]^T$ (Don't discretize in t.) Write down the resulting system of ordinary differential equations. Give computable expressions for every matrix and vector coefficient. (These coefficients will involve integrals. For full credit, give explicit expressions for the tightest possible limits of integration. In other words, don't integrate over intervals where you know the function is zero.)