AMSC/CMSC 661 Quiz 1 , Spring 2010

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no calculators, cellphones, or any other electronic devices, and don't communicate with other students. You may use the Larsson&Thomèe textbook, anything taken from the course website, and your own notes.

Name _____

1. (10) Consider the differential equation

$$\mathcal{A}u \equiv -(au')' + bu' + cu = f \text{ in } \Omega = (0,1)$$

$$u(0) = 0, \;\; u(1) = 0,$$

where the functions $a(x) > 2, c(x) \geq 0, b(x)$, and $f(x)$ are smooth on the interval $x \in [0,1]$. How sensitive is the solution to small changes in the boundary conditions? Specifically, if we change the problem to $\mathcal{A}v = f$ with boundary conditions $v(0) = \epsilon_1 > 0$ and $v(1) = \epsilon_2 > 0$, for small numbers ϵ_1, ϵ_2, how much does v differ from u? Justify your answer.

(This situation could arise in modeling a flat plate in thermal equilibrium except that it is difficult to maintain the boundary at a fixed temperature.)

2. (10 points) consider the problem

$$-u''(x) - u'(x) = e^{1-x},$$

$$u(0) = 0, \;\; u(1) = 1.$$

Use the maximum principle and the minimum principle, to tell me as much as you can about the solution to this problem, without actually solving the problem.