AMSC/CMSC 661 Quiz 6 , Spring 2010

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no calculators, cellphones, or any other electronic devices, and don't communicate with other students. You may use the Larsson&Thomèe textbook, anything taken from the course website, and your own notes.

Name _____

1. (10) Let $\Omega = \{\boldsymbol{x} : -1 \le x_1 \le 1, \ -1 \le x_2 \le 1\}$, and let $\Gamma(\Omega)$ be the boundary of Ω. Let

$$q(\boldsymbol{x}, t) = 3(x_1 - 1)(x_1 + 1) + 2t(x_2 - 1)(x_2 + 1) - 6t - 2t^2.$$

Consider the problem

$$\begin{aligned} \frac{\partial u(\boldsymbol{x}, t)}{\partial t} - \Delta u(\boldsymbol{x}, t) = &\quad q(\boldsymbol{x}, t) &&\text{for } \boldsymbol{x} \in \Omega \subset \mathcal{R}^2, t \in \mathcal{R}_+ \\ u(\boldsymbol{x}, 0) = &\quad 0 &&\text{for } \boldsymbol{x} \in \Omega \\ u(\boldsymbol{x}, t) = &\quad 0 &&\text{for } \boldsymbol{x} \in \Gamma(\Omega), t \in \mathcal{R}_+ \end{aligned}$$

Give a bound on

$$\max_{0 \le t \le 5} \max_{\boldsymbol{x} \in \Omega} |u(\boldsymbol{x}, t)|.$$

2a. (5) Show that $z_j(x) = \sin(2\pi jx)$ (j a positive integer) satisfies the equation $-z_{xx}(x) = \lambda z(x)$, and $z(0) = z(1) = 0$. What is λ?

2b. (5) Consider the problem

$$\begin{aligned} u_t - u_{xx} &= 0 && \text{for } x \in [0,1], t \geq 0, \\ u(x,0) &= x(1-x) && \text{for } x \in \Omega \\ u(x,t) &= 0 && \text{for } x \in \Gamma(\Omega), t \in \mathcal{R}_+ \end{aligned}$$

Suppose we express the solution as

$$u(x,t) = \sum_{j=1}^{\infty} w_j(t) z_j(x)$$

Give a formula for $w_1(t)$.