AMSC/CMSC 661 Quiz 8 , Spring 2010

Show all work. You may leave arithmetic expressions in any form that a calculator could evaluate. By putting your name on this paper, you agree to abide by the university's code of academic integrity in completing the quiz. Use no calculators, cellphones, or any other electronic devices, and don't communicate with other students. You may use the Larsson&Thomèe textbook, anything taken from the course website, and your own notes.

Name ____

1. (10) Let $\Omega = \{\boldsymbol{x} : -2 \le x_1 \le 1, \; -1 \le x_2 \le 1\}$, and let $\Gamma(\Omega)$ be the boundary of Ω. Let

$$q(\boldsymbol{x}, t) = 3x_1 - 2t(x_2 - 1)(x_1 + 1) - 6t + 2t^2.$$

Consider the problem

$$\begin{aligned} \frac{\partial u(\boldsymbol{x}, t)}{\partial t} - \Delta u(\boldsymbol{x}, t) = &\quad q(\boldsymbol{x}, t) \quad && \text{for } \boldsymbol{x} \in \Omega \subset \mathcal{R}^2, t \in \mathcal{R}_+ \\ u(\boldsymbol{x}, 0) = &\quad 3x_1 && \text{for } \boldsymbol{x} \in \Omega \\ u(\boldsymbol{x}, t) = &\quad t x_1 x_2 && \text{for } \boldsymbol{x} \in \Gamma(\Omega), t \in \mathcal{R}_+ \end{aligned}$$

Give a bound on

$$\max_{0 \le t \le 5} \max_{\boldsymbol{x} \in \Omega} |u(\boldsymbol{x}, t)|.$$

2. (10) Suppose that the problem

$$\begin{aligned} -\Delta w &= f(\boldsymbol{x}) \qquad && \text{for } \boldsymbol{x} \in \Omega, \\ w(\boldsymbol{x}) &= 0 && \text{for } \boldsymbol{x} \in \Gamma(\Omega) \end{aligned}$$

has been discretized using piecewise linear finite elements (with basis functions ϕ_j) to obtain the linear system $\boldsymbol{A}\boldsymbol{w} = \boldsymbol{f}$. The $2 \times n$ matrix $\boldsymbol{P}$ contains the coordinates of the vertices of the triangles of the mesh, the vector $\boldsymbol{w}$ contains the values of the approximate solution at the vertices, and $f_j = (f, \phi_j)$. Also suppose that you are given a matrix $\boldsymbol{B}$, the mass matrix, with entries $b_{ij} = (\phi_i, \phi_j)$.

Now suppose we want to solve

$$\begin{aligned} u_t - \Delta u &= f(\boldsymbol{x}) \qquad && \text{for } \boldsymbol{x} \in \Omega, t \geq 0, \\ u(\boldsymbol{x}, 0) &= v(\boldsymbol{x}) && \text{for } \boldsymbol{x} \in \Omega \\ u(\boldsymbol{x}, t) &= 0 && \text{for } \boldsymbol{x} \in \Gamma(\Omega), t \in \mathcal{R}_+ \end{aligned}$$

Write Matlab code to obtain an approximate solution to this problem for $t = 0, 0.1, 0.2, \ldots 1.0$ using the backward Euler method in time and finite elements in space. (If this is too confusing, you can get up to 7 points credit for forward Euler.)