My research has centered upon several themes, primarily related to computational linear algebra, scientific computing, and optimization. The work has involved a mixture of algorithm development and scientific applications, drawing upon tools in applied mathematics, numerical analysis, and computer science. These themes have led to applications in science and engineering.

This work has been published in computer science and applied mathematics journals as well as journals of physics, chemistry, aerospace engineering, biology, medicine, and electrical engineering. The references in the following sections are numbered chronologically in three groups: journal publications \([J]\) (available at http://www.cs.umd.edu/users/oleary/pubj.html), conference proceedings \([C]\) (available at http://www.cs.umd.edu/users/oleary/pubc.html), and technical reports\([T]\) (available at http://www.cs.umd.edu/users/oleary/tr.html).
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1 Krylov Subspace Methods

In my thesis and in subsequent work, the effectiveness of the preconditioned conjugate gradient algorithm was demonstrated for discretizations of linear elliptic partial differential equations [C1], nonlinear elliptic equations [J1], and free boundary problems for linear and nonlinear elliptic equations [J8] [J3]. Application of the conjugate gradient algorithm to general quadratic programming problems was considered [T4]. This work was applied to the analysis of torsion on an elasto-plastic bar [J2] and water flow in an excavation site [C2].

Polynomial preconditioners for the conjugate gradient algorithm were studied in [J33].

A block form of the conjugate gradient algorithm, useful for solving multiple linear systems and for linear systems with specialized eigenvalue distributions, was developed and analyzed [J6]. The parallel implementation of the algorithm was studied [43] [J24] [J32].

The quasi-Newton family of algorithm was extended to block form in [J39], and new insights on Broyden’s method applied to linear systems were developed [J41]. Efficient use of conjugate gradient algorithms for computing the search directions in interior point methods was studied in [J55].

A literature review of the first 25 years of the conjugate gradient algorithm was published in 1989, jointly with Gene Golub [J28] and an updated overview is given in [H2].

Stagnation of the GMRES algorithm for solving nonsymmetric systems of equations was studied in [J64].

Zdeněk Strakoš, Petr Tichý, and I contributed to the understanding of the convergence of Krylov methods when implemented on computers in inexact arithmetic [J81], by exploiting the relation of these methods to Gauss quadrature.

[C1] (Invited paper) Paul Concus, Gene H. Golub, and Dianne P. O’Leary,
“A generalized conjugate gradient method for the numerical solution of elliptic partial differential equations,”


2 Optimization

Optimization problems arising from partial differential equations can lead to linear programming [C2], quadratic programming [J2], [J8], or more general optimization problem [J3].

I developed the discrete Newton method [J12], today called the truncated Newton method, for use when derivatives are not easy to calculate.

Other work concerned understanding quasi-Newton methods [J41] and making them more efficient [J47].

[J54] is a survey of the impact of numerical linear algebra on optimization.

Efficient adaptive use of conjugate gradient algorithms for computing the search directions in interior point methods was studied in [J55].

In [J83], Haw-ren Fang and I reviewed modified Newton methods based on the Cholesky factorization, determining the properties of existing methods and deriving new methods that perform better.

Jin Hyuk Jung and I proposed efficient algorithms for solving linear programming problems on inexpensive parallel computers GPUs [J89], for training support vector machines [J90], and for solving convex quadratic programming problems with many constraints [J95].

Simon Schurr, Andre’ Tits, and I have posed two problems related to conic convex optimization. First [J75] we studied conditions under which such problems are well posed, in the sense that solutions can be constructed for arbitrary specifications of the data values. Second, we studied the accuracy necessary to evaluate the functions in order to preserve polynomial complexity in the solution algorithms [J93].


3 Eigenproblems and Matrix Studies

A discussion during an informal seminar led to a proposal for a quick estimation of the largest eigenvalue of a matrix [5]. A minor improvement on condition number estimation was given in [7]. The accurate computation of eigenvalues of arrowhead matrices was considered in [31]. Pete Stewart and I also studied the use of a modified Rayleigh quotient iteration for finding eigenvalues [49].

Matrix scaling was studied in [65], and factorizations of symmetric tridiagonal and triadic matrices were studied in [78], for later use in modified Newton methods for optimization.
Markov Chains are used to model processes such as behavior of queueing networks. Both the short term behavior (e.g., mean first passage times) and the long term behavior (e.g., stationary vector) are of interest, and this work has focused on both of these problems.

The basic computational problem for Markov chains is determining the stationary vector, defining the long-term behavior of the chain. Grassman, Taksar, and Heyman proposed an algorithm that O’Cinneide has shown to compute an approximation to the stationary vector with low relative error in each component. Jason Wu and I developed a block form of the GTH algorithm, more efficient on high performance architectures, and showed that it, too, produces a vector with low relative error. We demonstrated the efficiency
of the algorithm on vector processors and on workstations with hierarchical memory [J42]. Iterative methods for finding stationary vectors were studied in [C9].

A great deal of attention has been devoted to computing stationary vectors, but less attention has been given to computational algorithms for other parameters associated with these chains. Daniel Heyman (Bellcore) and I developed and evaluated algorithms for computing the fundamental matrix, the group generalized inverse, and the mean and variance of first passage times for discrete time regular Markov chains [C14] and then studied ill-conditioned problems arising in this area [J44].


5 Robust Regression

The development of computational algorithms for iteratively reweighted least square problems [J30] led to an interesting problem concerning the uniform boundedness of scaled projectors [J29] and to further investigation of structured problems, with applications to image deblurring [J80].
6 Numerical Solution of Ill-Posed Problems

In ill-posed problems, small changes in the data can cause arbitrarily large changes in the results. Although it would be nice to avoid such problems, they have important applications in medicine (computerized tomography), remote sensing (determining whether a nuclear reactor has a crack), and astronomy (image processing).

Two projects were performed jointly with workers at the National Institute of Standards and Technology (formerly the National Bureau of Standards). In the first, data related to non-destructive detection of crack formation was analyzed by filtering techniques using a small problem-dependent set of basis functions [J11]. This was the first hybrid algorithm for ill-posed problems: the given problem was projected to a subspace and regularization was applied there.

In the second project, confidence intervals for spectroscopy data were computed using a nonnegativity constraint on the solution [J21] [J38].

Other work focused on the choice of optimization criteria for ill-posed problems [J37], studying the characteristics of solutions produced by various regularization methods, including truncated least squares, regularized least squares, regularized total least squares, and truncated total least squares [C11] [C15] [J43] [J51]. Efficient numerical algorithms for computing these solutions in image processing applications were also proposed [J50] [J45] [C16] [J61].

The regularization of discretized problems by iterative methods was studied
with Misha Kilmer [J57], demonstrating that regularization could be provided easily if the iterative method was viewed as projecting the continuous problem into a lower dimensional subspace. Choice of the regularization parameter was considered in [J59], [J85], and [J87].


7 Parallel Algorithms

G.W. Stewart and I proposed the use of the data-flow model for developing fine- and medium-grained algorithms for dense matrix problems on message-passing parallel machines; the computational model and several sample algorithms were developed [J19], and a detailed analysis of a parallel Cholesky
algorithm was given [J22]. A similar analysis was given for the block conjugate gradient algorithm [J24].

Some “coloring algorithms” were developed which make parallel iterative methods more efficient for certain network problems, including discretizations of elliptic differential equations [J18]. A different class of parallel iterative methods was developed in joint work with R.E. White, which proposed and analyzed the simultaneous use of several matrix splittings [J20]. These multisplitting algorithms have since been the subject of considerable work by other researchers and formed the basis for the preconditioning algorithms used in [C7] [J36].

More recently, I have worked on GPU algorithms [J89].


8 Parallel Architectures and Systems

In the course of collaborative experimental work on parallel algorithms in the early 1980’s, deficiencies of existing software systems led to the development of a multi-tasking and portable communication system [C4] and to the proof that its underlying principle led to deterministic computation [J26]. The deficiencies of hardware systems spurred the development of some specialized systolic arrays for matrix data movement [J23] and the design of a hybrid machine, capable of shared-memory interaction for neighboring processors and efficient message passing for more distant processors [T8].


9 Image Processing

This work, joint with various experts in image processing, involves the adaptation of techniques in linear algebra and optimization. Projects included using the singular value decomposition for classifying images [J9], applying function minimization methods to noise smoothing and edge reinforcement [J13] [J14], using multi-level iterative methods for function minimization [J15], and analyzing convergence of iterations used in image processing [J17].

An efficient algorithm for image compression was also developed, making use of linear algebra and discrete optimization techniques [J16], and several algorithms were studied for approximating two dimensional convolution operators by a product of convolutions with smaller support [J27].

Recent work has focused on the solution of the ill-posed problems arising in deblurring. Various optimization criteria have been evaluated [C12], and J. G. Nagy and I have developed algorithms that are efficient when the point spread function (the blurring function) is spatially variant, as in the Hubble Space Telescope [C16],[J45]. We have also worked on computing and displaying confidence intervals for the reconstructed images [J61]. This work was extended to robust regression in [J80]. Armin Pruessner and I studied blind deconvolution, in which the blurring function as well as the true image is to be determined [J63], and the structure of the blurring matrix was exploited in [J67] and [J74].

An application to Ladar images was made in [J66].

Some of this work is summarized in a monograph [B1], written at the level of an advanced undergraduate or beginning graduate student, designed to motivate mathematics and computer science students to learn about computational methods.


10 Signal Processing and Control

Collaborators in aeronautical engineering were interested in the numerical solution of a constrained Sylvester equation, with applications in control theory, and we developed an efficient algorithm [C5] [C6] [C8] [J34] [J35].

An efficient variant of the ESPRIT algorithm for determining direction of arrival of signals reaching an array of sensors was developed [C10] [J40].


11 Information Retrieval

The semi-discrete decomposition, developed with Shmuel Peleg for image compression, has proved quite useful in latent semantic indexing, a method of document retrieval [C17] [J48].

Methods for document summarization based on hidden Markov models and matrix decompositions are studied in [J62]. We demonstrated the success of the methods for summarizing medical documents in [C26]. Our methods have been quite successful in the DUC (Document Understanding Conference) and TREC competitions [C20],[C21],[C22],[C25],[C27],[C28],[C29] and recently they performed as well as human summarizers in an evaluation on summarizing multi-lingual document sets [C30]; this shows that our summarizer is quite good, but also that the evaluation metrics are quite primitive! Further information about our summarization work is available in [C31],[C32],[C33],[C34].
A full retrieval system that processes a query, clusters the resulting documents, and creates summaries of each cluster is presented in [J82] and available at http://stiefel.cs.umd.edu:8080/qcs/


12 Quantum Computing

Quantum computers may offer a way to solve certain problems that are larger and more complex than any that can be solved on conventional computers. One way to view a (gated) quantum computer is as a machine that multiplies a vector by a unitary matrix. The number of possible data values equals the dimension of the vector, and the absolute value of the $i$th component of the vector represents the probability that the data or the answer is equal to the $i$th value. The unitary matrix is designed to make the absolute value of the entry corresponding to the correct answer quite close to one.

Stephen Bullock, Gavin Brennen, and I investigated several questions relevant to the decomposition of the unitary matrix into quantum “gates” that can actually be implemented in hardware. In [J69], we used a matrix decomposition to construct these gates. In [J70], [J71] and [J76], we used controlled Householder gates to implement circuits for qudits, quantum variables that can take on $d > 2$ values rather than the traditional 2 values used for qubits. Some Givens and Householder gates are cheaper than others, depending on the components they access, and we determined a systematic way to determine whether a set of rotation planes is sufficient in [J72]. Then we considered how much such quantum operations might be sped up [J79], for example by using more than one pair of lasers to excite multiple transitions among the hyperfine states of the atomic alkalies.

Another possible mechanism for quantum computing is the use of adiabatic systems. In [J86], Michael O’Hara and I investigated the effects of perturbations on such systems.

Finally, O’Hara and I studied the unexpectedly large ground-state energy gaps in a certain class of Hamiltonians [J92].


13 Physics Applications

Helmholtz equations are used to model a variety of important physical systems, ranging from heat distribution to the transmission of sound. Olof Widlund and I developed efficient algorithms for solving the Helmholtz equation on general three dimensional regions with Dirichlet or Neumann boundary conditions, imbedding the region in a cube. Innovations involved the proof of existence of the discrete solution, development of effective scaling strategies, and the choice of effective storage structures [J4] [J10]. Efficient variants of these algorithms for problems with mixed boundary conditions over a union of rectangles were later developed [J25], with application to a National Bureau of Standards (now National Institute of Standards and Technology) model of smoke transport in buildings.

Later, Howard Elman, Oliver Ernst, and I considered the difficulties encountered when the Helmholtz parameter is negative, leading to indefinite systems of linear equations. Results are presented in [J46] [J52] [J60]. These problems arise in studying wave phenomena, for example, transmission of sound underwater. In collaboration with post-doc Michael Stewart, we extended our study to problems in which the boundary conditions are stochastic [J68].

A method for solving an important physics problem, approximating the number of monomer-dimer coverings in periodic lattices, was given in [J58]. This model has a variety of uses in solid state physics, ranging from studying spontaneous magnetization to phase transitions in multicomponent liquids and biological membranes.

The behavior of systems such as superconductors are described using Schrödinger’s equation. In [J86], Michael O’Hara and I considered the effects of perturbations on the behavior of solutions to this equation.


structuring, and mixed or Neumann boundary conditions,” Applied Nu-

[J46] Howard C. Elman and Dianne P. O’Leary “Efficient Iterative Solution
of the Three-Dimensional Helmholtz Equation,” Journal of Computational

[J52] Howard C. Elman and Dianne P. O’Leary, ”Eigenanalysis of Some Pre-
231-257.

[J58] Isabel Beichl, Dianne P. O’Leary, and Francis Sullivan, “Approximat-
ing the Number of Monomer-Dimer Coverings in Periodic Lattices,”

[J60] Howard C. Elman, Oliver G. Ernst, and Dianne P. O’Leary, “A Multi-
grid Method Enhanced by Krylov Subspace Iteration for Discrete Helmholtz

[J68] Howard C. Elman, Oliver G. Ernst, Dianne P. O’Leary, and Michael
Stewart, “Efficient Iterative Algorithms for the Stochastic Finite Ele-
ment Method with Application to Acoustic Scattering,” Computer

20 pages. http://link.aps.org/abstract/PRA/v77/e042319 DOI:
10.1103/PhysRevA.77.042319. Chosen for inclusion in Virtual Journal of
Applications of Superconductivity 14:9 (2008) and Virtual Journal of

14 Medical, Biological, and Chemical Appli-
cations

In addition to work in medical informatics, I have worked on medical image
processing. Numerical methods for classifying cytology specimens (e.g., pap
smears) were compared in [J9]. In collaboration with medical researchers, methods for identifying chromosomes were developed in [J53] and [C23].

The structure of a protein provides critical information in determining its function. A new algorithm for determining the 3-dimensional configuration of proteins was described in [J73], and its use was demonstrated on a backbone model of proteins. In [J77] and [J84] we proposed two fast screening algorithms for finding proteins whose structure is expected to be similar to one presented for classification. The relation between the structure of the connectivity graph and the essentiality of a protein in its interaction network was investigated in [J88].

Konstantin Berlin, David Fushman, and I studied the use of nuclear magnetic resonance (NMR) measurements in determining protein shape. First we developed a fast algorithm for predicting residual dipolar couplings [J94], and then we used it to determine structure of docked complexes [J96].


15 Software

My publicly available software includes an algorithm for solving the Helmholtz equation for the Dirichlet problem on general bounded three dimensional regions [J10], an algorithm to compute the Semidiscrete Matrix Decomposition [J56], and a system for processing queries (as, for example, Google might), cluster the potentially relevant documents, and return a summary of each cluster [J82].
16 History of Scientific Computing

An interest in the history of scientific computing in the second half of the 20th century has led to many publications, listed here.


[H11] Lars Eldén, Misha E. Kilmer, and Dianne P. O’Leary, “Updating and Downdating Matrix Decompositions,” Chapter 5 in *G. W. Stewart:


17 Current work

Current funding supports work on ill-posed problems and nonlinear programming.

As always, research directions are determined by a mixture of planning and serendipity, with motivation from applications problems as well as open questions in applied mathematics and numerical analysis.