Due at the start of class Tuesday, Feb 17, 2004.
Read Chapters 1 and 2 from Harel's book.

Problem 1. Give an instance of the stable marriage problem where there are multiple stable marriages. Show at least two different stable marriages and show how you computed them.

Problem 2. You are given as input a sequence of $2n$ integers. Design an algorithm that partitions the numbers into n pairs with the property that the partition minimizes the maximum sum of a pair. For example, if the numbers are $(1, 9, 5, 3)$ the answer is $(1, 9)$ and $(5, 3)$ with maximum value 10. Write down a short argument that justifies why you think your algorithm works.

Problem 3. Prove (by induction) that $f_{i+2} \geq \phi^i$, where $\phi = \frac{(1+\sqrt{5})}{2}$. This shows that the Fibonacci numbers grow at an exponential rate.

Problem 4. Prove by induction that

$$\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}.$$

Problem 5. Give an example of the Hotel partner problem where there is no stable solution. Recall that the problem is defined as follows. There are $2n$ people, and n hotel rooms. Each person rank orders the remaining $2n-1$ people in preference order. We have to assign two people to a room with the property that the assignment is stable. An unstable assignment is one where there is a pair of people assigned to different rooms, such that they both prefer each other to the current partners.