Homework 4
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Problem 1

$N$ nodes share a 1 Mbps broadcast channel using the slotted ALOHA multiple access protocol. A “central” station (not one of the $N$ nodes) is responsible for acking successful transmissions and indicating slot boundaries. As usual with slotted ALOHA, a node can hear only the central station but the central station can hear every node.

Each node always has a packet to transmit. All packets are 1 KB in size, with the first byte identifying the sender. Each node transmits in a slot with probability $p$.

At the end of a slot, the controller station sends a short “ack” message indicating the sender of the successful transmission, if there was one. The ack message also serves to indicate the start of the next slot. Assume that the ack message is very short, so that the size of a slot is essentially the time to transmit a 1 KB packet.

Let $G$ be the goodput in packets per slot.
Let $F$ be the goodput in packets per second.

a. Express $G$ in terms of $N$ and $p$.
b. What is the optimal $p$ in terms of $N$, i.e., for any $N$, the $p$ that yields the highest $G$.
c. Express $G$ in terms of $N$ for the optimal $p$.
d. Express $F$ in terms of $N$ for the optimal $p$.
e. Obtain $F$ for $N = 4$ for the optimal $p$. Your answer must be a decimal number.

Problem 2

As in problem 1, $N$ nodes and a central station share a 1 Mbps broadcast channel. Each node always has a packet to transmit. All packets are 1 KB in size.

But unlike problem 1, the nodes use a hybrid of slotted ALOHA and reservation. Specifically, the nodes use slotted ALOHA with 100$\mu$s slots to transmit small “reservation” messages. Each station transmits in a slot with probability $p$. When a node successfully transmits its reservation message (i.e., gets an ack from the central station), it then sends a 1 KB packet (without any possibility of collision).

Thus the activity on the channel is an alternating sequence of contention intervals and packet transmissions, where each contention interval consists of one or more 100$\mu$s slots with only the last one being successful.

It turns out that the average number of slots in a contention interval is $1/A$, where $A$ is the probability of a successful transmission in a slot.

Let $F$ be the goodput in packets per second.

a. Express $F$ in terms of $N$ and $p$.
b. What is the optimal $p$ in terms of $N$, i.e., for any $N$, the $p$ that yields the highest probability of success in a slot.
c. Obtain $F$ in terms of $N$ for the optimal $p$.
d. Obtain $F$ for $N = 4$ for the optimal $p$. Your answer must be a decimal number.
Problem 3

Consider an Ethernet with three nodes, A, B, and C. Let the jam signal be \( x \) seconds in duration. Assume that the propagation delays are small compared to \( x \). Hence a collision lasts for \( x \) seconds, and all nodes see the start (and end) of a collision at the same time. (Make sure you understand why this is so.)

At time 0, each node has 1 packet to send. Each packet transmission time is 4 slots (i.e., \( 4x \) seconds). The random delays chosen by the nodes are as follows:

\[
\begin{align*}
A &: \ 3, 5, 7, 8 \\
&\text{(i.e., the first backoff delay is 3, the next is 5, and so on)} \\
B &: \ 3, 6, 4, 9 \\
C &: \ 4, 3, 7, 6
\end{align*}
\]

Draw a chart indicating the transmissions (successful and collisions) of the three nodes versus time. Indicate when each packet is successfully sent. A skeleton is shown below to get you started.