# Object Transfer using Path Reversal: Distributed Path-Reversal Algorithm

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# Outline

# Path reversal algorithm



- Systems attached to a fifo channel; obj initially at a0
- Messages
  - [REQ,j]: request msg // j is issuer (not forwarder)
  - [OBJ]: object-carrying msg // ignore value for now
- System j: eating (has obj), hungry (wants obj), thinking (o/w)
- System j has a "last" pointer
  - addr in the last req msg rcvd by j after last becoming hungry.
  - nil if no such msg
  - initially nil at a0, and a0 elsewhere
- System j has a "next" pointer
  - nil if j thinking or not rcvd req since non-thinking
  - o/w equals addr in the first req msg rcvd since non-thinking
  - initially nil

# System j rules



- become hungry only if thinking: // $H$(j)
  send [REQ,j] to lst
  set lst to nil
- rcv [OBJ]: // $E$(j)
  become eating
- become thinking only if eating and nxt non-nil: // $T$(j)
  send object to nxt
  set nxt to nil
- rcv [REQ,k]: // $R$(j,k)
  if lst not nil
    send [REQ,k] to lst
    set lst to k
  else set nxt and lst to k

# Conventions



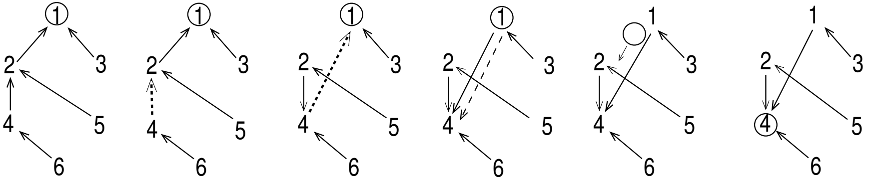
- j–k is a last edge: `j.lst` is not nil and equals k
- j–k is a next edge: `j.nxt` is not nil and equals k
- j–k is a request edge: message `[REQ,j]` is in transit to k

- digraph: directed multi-graph
- *LNR*: digraph [ addresses; last/next/request edges ]
- *L*: digraph [ addresses; last edges ]
- *LR*: digraph [ addresses; last/request edges ]

- Drawing conventions
  - last edges: ———
  - next edges: - - - - -
  - request edges: · · · · · ·

# Serial evolution: at most 1 hungry at a time



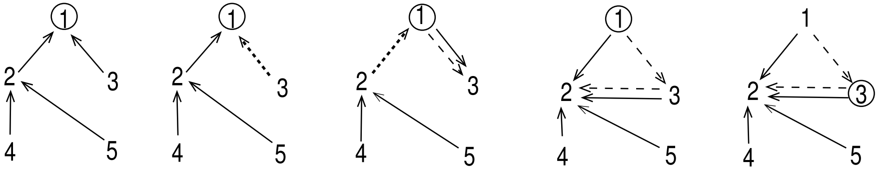



initially 1 eating | 4 turns hungry, sends [REQ,4] to 2 | 2 forwards [REQ,4] to 1 | 1 rcvs [REQ,4] | 1 turns thinking sends obj to 4 | 4 rcvs object, turns eating

- $L$ is an in-tree when no req msg in transit
- each request effects a path reversal
  - j's req travels from j to root
  - all nodes on path now point to j
- amortized cost of $\log N$

# Serial evolution: more than 1 hungry at a time






initially
1 eating

3 turns hungry,
sends [REQ,3].

1 rcvs [REQ,3].
2 turns hungry,
sends [REQ,2]

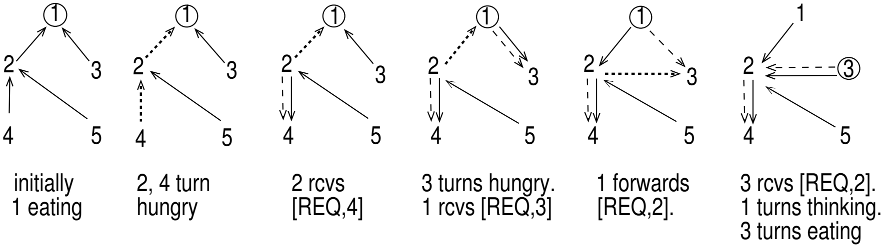
1 frwrds [REQ,2].
3 rcvs [REQ,2]

1 turns thinking
sends object.
3 turns eating

- $L$ evolves as before, so amortized cost same
- next ptrs form queue

# Non-serial evolution



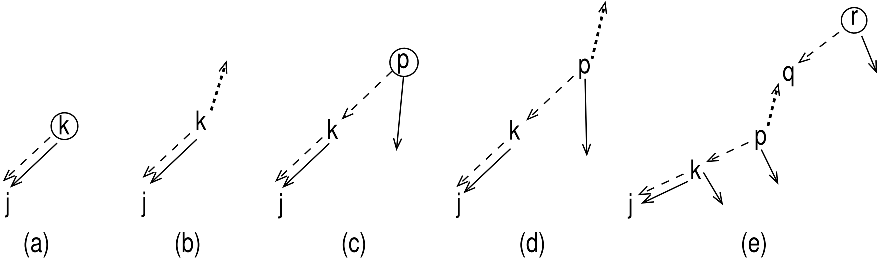



- $L$ may never be an in-tree
- can be several next-ptr queues
- progress?
- amortized cost?

# Does hungry j eventually eat?



- Possibilities when j's request msg dies




- Above has implicit assumptions
  - *L* remains acyclic
  - j.nxt never points to j
  - ...

- Now to make argument rigorous

# Outline

# Basic safety properties – 1



- $Inv\ A_1$–$A_3$ holds // via inv rule

  $A_1$ : forone(j: either (j eating) or (obj in transit to j))

  $A_2$ : j.nxt ≠ nil ⇒ (j.lst ≠ nil and (j not thinking))

  $A_3$ : (j thinking) ⇒ j.lst ≠ nil

- $Inv\ B_1$–$B_2$ holds // via inv rule assuming $Inv\ A_1$–$A_3$

  $B_1$ : *LR* has exactly 1 undirected path between every two nodes

  $B_2$ : forall(j: j.lst ≠ j)

# Basic safety properties – 2

- $\mathit{Inv}\ B_3$–$B_5$ holds // via inv rule assuming $\mathit{Inv}\ A_1$–$A_3$, $B_1$–$B_3$

$B_3$ : forall j: exactly 1 of the following holds

- j thinking or
- [REQ,j] in transit or
- forsome (k: k.nxt = j) or
- [OBJ] in transit to j or
- j eating

$B_4$ : forall (j: at most one [REQ,j] in transit)

$B_5$ : forall (j: j.nxt $\neq$ j and num(k: k.nxt = j) $\leq$ 1)

# Priority digraph *Pr*



- Want a digraph *Pr* that captures relative priorities of nodes
- Want j–k in *Pr* to mean j has lower priority than k
  - j–k is a pr-next edge: k–j is a next edge
  - j–k is a pr-last edge: j–k is a last edge and j thinking
  - j–k is a pr-request edge: j–k is a request edge
- *Pr*: digraph [ addresses; req/pr-next/pr-last edges ]
- Define
  - pr-path: directed path in *Pr*
  - j pr-reachable from k: pr-path from k to j
  - lr-path: undirected path in *LR*

# Some safety properties of *Pr*



- $Inv\ C_1$–$C_3$ // via inv rule assuming $Inv\ A_1$–$A_3$, $B_1$–$B_5$

$C_1$ : (*Pr* in-tree) and
(*Pr*'s root eating or obj in transit to it)

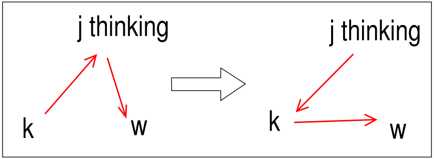
$C_2$ : pr-path from k to `j` $\Rightarrow$
forall x on the lr-path between `j` and k:
pr-path from x to `j`

$C_3$ : (`j` not thinking) and `j.lst` $=$ `k` $\neq$ `nil` $\Rightarrow$
((pr-path from k to j) and (k hungry))

# Proof of $Inv\ C_1$ – 1



- Initially $Pr$ is the same as $LR$, so $C_1$, $C_2$, $C_3$ hold.
- j starts eating: $Pr$ not affected, so $C_1$ preserved
- j issues req when `j.lst`=`w`:
  `j`–`w` goes from pr-last edge to pr-req edge. $C_1$ preserved
- j rcvs req k when thinking:
  j–w, k–j $\longrightarrow$ k–w, j–k.
  $C_1$ preserved (# edges, connectivity preserved)



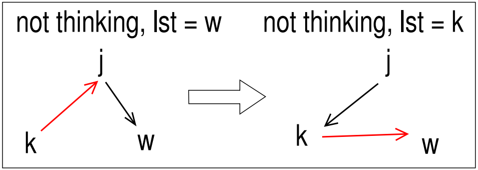

- j rcvs req k when not thinking, `j.lst`=nil:
  k–j goes from pr-req edge to pr-next edge. $C_1$ is preserved

# Proof of $Inv\ C_1$ – 2



- j rcvs req k when not thinking, `j.lst`$=$`w`$\neq$ nil:
  - k–j replaced by k–w.

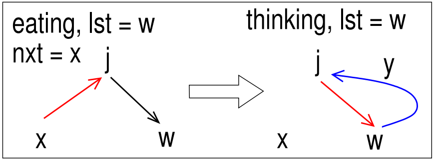
    (j–w, j–k are not in *Pr*)




  - # edges preserved, so suff to show connectivity preserved
  - old *Pr* has pr-path(w,j) // from $C_3$
  - suff if old *Pr* has no pr-path(w,k)
  - assume old *Pr* has pr-path(w,k)
    - all nodes on lr-path(k,w) have pr-path to k // from $C_2$
    - so lr-path(k,w) avoids lr-edges k–j and j–w // from $C_1$, $C_2$
    - so undirected cycle in old *LR* // negates $B_1$

# Proof of $Inv\ C_1$ – 3

- j stops eating when j.nxt=x, j.lst=w:
  - x–j replaced by j–w




  - old *Pr* in-tree/root j; to show new *Pr* in-tree/root x
  - suff if old *Pr* has pr-path(w,x); assume not so
  - so old *Pr* has pr-path(w,y) and pr-edge y–j, where y≠x
    - y–j is also a lr-edge
    - if y=w, then old *LR* has cycle [y,j,y] // negates $B_1$
    - if y≠w, then lr-path(y,w) avoids j // $C_1$, $C_2$
      lr-path and lr-edges y–j, j–w form lr-cycle // negates $B_1$

# Outline

# Progress metric in *Pr*

- *Pr* can have several next-edge paths
- next-queue: a maximal next-edge path // wff coz *Pr* in-tree

  tail $\bullet \leftarrow \cdots \leftarrow \mathtt{j} \leftarrow \cdots \leftarrow \bullet$ $hd_{\mathtt{j}}$

- $hd_{\mathtt{j}}$: head of j's next-queue
  - j if j has no incoming next edge
- Goal: fn $F(\mathtt{j})$ st
  - increases while req $hd_{\mathtt{j}}$ in transit
  - has upper bound at which $hd_{\mathtt{j}}$ has obj (and no req msg)

# Progress: $\alpha_{\mathtt{j}}$



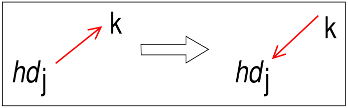
- Consider $\alpha_{\mathtt{j}}$: set of nodes with pr-paths to $hd_{\mathtt{j}}$
- Following hold

$D_1$ : $\alpha_{\mathtt{j}}$ increases when req $hd_{\mathtt{j}}$ is rcvd by a system that is thinking or whose last pointer is nil

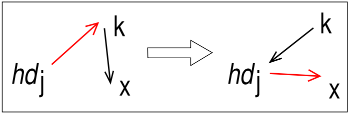
$D_2$ : $\alpha_{\mathtt{j}}$ does not decrease while j is hungry

# Proof of $D_1$

- Let req $hd_j$ be rcvd by k
- Prior to rcv, k $\notin \alpha_j$ // *Pr* in-tree, has req edge [$hd_j$, k]
- Different cases of k
  - k thinking:
    pr-req $hd_j$–k $\rightarrow$ pr-last k–$hd_j$
    $\alpha_j \uparrow$ by $k^+$




  - k not thinking, k.1st nil:
    pr-req $hd_j$–k $\rightarrow$ pr-next k–$hd_j$ // as in above figure
    k becomes $hd_j$, $\alpha_j \uparrow$ by $k^+$
  - k not thinking, k.1st = x $\neq$ nil:
    pr-req $hd_j$–k $\rightarrow$ pr-req $hd_j$–x
    $\alpha_j$ no change



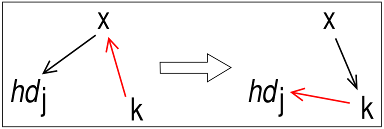
# Proof of $D_2$

- Consider steps other than rx of req $hd_\mathtt{j}$
- z starts eating: neither Pr nor $\alpha_\mathtt{j}$ change
- z issues a request: pr-last z–. → pr-req // $\alpha_\mathtt{j}$ same
- z not in $\alpha_\mathtt{j}$: does not decrease $\alpha_\mathtt{j}$
- z in $\alpha_\mathtt{j}$ sends object to y:
  old *Pr*: z is $hd_\mathtt{j}$, *Pr*-root
  new *Pr*: y is $hd_\mathtt{j}$, *Pr*-root // $\alpha_\mathtt{j}$ same (at max)
- z in $\alpha_\mathtt{j}$ rcvs req k when z.lst $=$ x $\neq$ nil:
  pr-req k–j → pr-req k–x,
  old *Pr* has pr-path(z,x) (from $C_3$) // $\alpha$(j) does not decrease
- z in $\alpha_\mathtt{j}$ rcvs req k when z.lst nil: <do it>

# Progress: $\beta_j$



- To compensate for $\alpha_j$, want fn $\beta_j$ that
  $X_1$ : $\uparrow$ when req $hd_j$ rcvd by non-thinking k with non-nil 1st
  $X_2$ : $\downarrow$ only if $\alpha_j \uparrow$ simultaneously
- Consider $\beta_j$: set of non-thinking nodes whose 1st equals $hd_j$
- $\beta_j$ and $\alpha_j$ are disjoint // pr-path from $hd_j$ to $\beta_j$ (from $C_2$)
- $X_1$ holds because rcv adds k to $\beta_j$
- $X_2$ holds. x leaves $\beta_j$ in only two ways:
  - x starts thinking: creates pr-last x$-hd_j$, so $\alpha_j \uparrow$ by x
  - x rcvs req k:
    pr-req k$-$x $\rightarrow$ k$-$hd(j)
    so $\alpha_j \uparrow$ k
    (k $\notin$ old $\alpha_j$ ($C_1$–$C_2$))



- So $F_{\mathtt{j}} = [\alpha_{\mathtt{j}}.\mathtt{size}, \beta_{\mathtt{j}}.\mathtt{size}]$ under lexicographic ordering works
- We have established the following ($D_5$ used in serializability):

$D_3$ : (j eating and k hungry) *leads-to* j.nxt ≠ nil

$D_4$ : ((j eating and j.nxt ≠ nil)) *leads-to* j.nxt = nil)
⇒ (k hungry *leads-to* k eating)

$D_5$ : ((j and k are hungry) and (j pr-reachable from k))
*unless* ((j eating) and (k hungry))

# Outline

# Serializability – 1



- Goal: Transform any finite evolution $x$ via commutations to a serial evolution $y$ with the same set of sends and rcvs
- Let p do the $i$th eating step in $x$, and q do the preceding one.

  The $i$th eating step is the culmination of
  - one $H(p)$ step ($p$ becomes hungry)
  - one or more $R(., p)$ steps (rcv req $p$)
  - one $T(q)$ step ($q$ starts thinking)
  - one $E(p)$ step ($p$ starts eating)

  Let $v_i$ be the sequence of the above steps
- Let $w$ be the sequence of $x$-steps not in any $v_i$
- $x$ is a merge of $v_1, v_2, \cdots, w$
- Will show that $y$ is $v_1 \circ v_2 \circ \cdots \circ w$ // hence same cost

- Lemma 16.1: Let $f$ and $g$ be two successive steps in $x$ st
  - $f$ belongs to $v_i$ and
  - $g$ belongs to $v_j$, $j > i$, or to $w$

  Then $f$ and $g$ commute wrt the msgs sent and rcvd
- $x$ can be transformed to $y$ by repeatedly applying lemma 16.1
- Proof of lemma follows

# Proof of Lemma 16.1 – 1

- Let $g$ be $H$ or $R$ of $v_i$, involving req p.
  Let $f$ be $H$ or $R$ of $v_j$, involving req q.
  - Let $g$ rcv msg sent by $f$. Then $g$ in $v_j$ // contradiction
  - Let $f$ and $g$ be of same node x.
    Then pr-path(q,p) just after $f$.
    So p eats before q (from $D_5$). // contradiction

  Hence $f$ and $g$ commute, preserving sends and rcvs

- Let $g$ be $H$ or $R$ of $v_i$, involving req p.
  Let $f$ be $H$ or $R$ of $w$.
  - same as above case

# Proof of Lemma 16.1 – 2



- Let $g$ be $E$ of $v_i$, ie, rcv obj.
  Let $f$ be $H$ or $R$ of $v_j$ or $w$, ie, rcv req.
  - $g$ rcvs obj and $f$ sends req. So $g$ does not rcv from $f$.
  - Let $f$ and $g$ be at the same system.
    Req rcv step ($f$) is same whether hungry $(f, g)$ or eating $(g, f)$.
    So $f$ and $g$ can be interchanged.

  Hence $f$ and $g$ commute, preserving sends and rcvs

# Proof of Lemma 16.1 – 3

- Let $g$ be $T$ of $v_i$, ie, send obj.
  Let $f$ be $H$ or $R$ of $v_j$ or $w$.
  - $g$, being a $T$, does not rcv from $f$
  - Let $f$ and $g$ be at the same system, say x.
    Then $f$ cannot be $H$ (o/w $g$ could not be $T$)
    Thus $f$ is a $R$ step.
    - Suppose x.1st was nil prior to $f$.
      Then $g$ would send obj in response to $f$,
      so $f$ belongs to $v_i$ // contradicts $j > i$.
    - Suppose x.1st was non-nil prior to $f$.
      Then $f$ and $g$ commute because req rcv $(f)$
      same whether eating $(f, g)$ or thinking $(g, f)$.